Essays on Macroeconomic Theory as a Guide to Economic Policy

DISSERTATION

zur Erlangung des akademischen Grades
doctor rerum politicarum
(Dr. rer. pol.)
im Fach Volkswirtschaftslehre

eingereicht an der
Wirtschaftswissenschaftlichen Fakultät
Humboldt-Universität zu Berlin

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eingereicht am:	14. April 2009
Tag des Kolloquiums:	10. Juli 2009

Abstract

This dissertation consists of four chapters and an appendix. After an introductory chapter with an extended literature overview, Chapter 2 is dedicated to the topic of macroeconomic policy in a heterogeneous monetary union. Chapter 3 investigates the macroeconomics of real estate, and Chapter 4 deals with a New Keynesian open economy model versus the six major puzzles in International Macroeconomics. An appendix provides some derivations and mathematical details. In the following, I briefly sketch the approach and main findings of Chapters 2 to 4.

Chapter 2 uses a two-country model with a central bank maximizing union-wide welfare and two fiscal authorities minimizing comparable, but slightly different country-wide losses. The rivalry between the three authorities is analyzed in seven static games. Comparing a homogeneous with a heterogeneous monetary union, welfare losses relative to the social optimum are found to be significantly larger in a heterogeneous union. The best-performing scenarios are cooperation between all authorities and monetary leadership. Cooperation between the fiscal authorities is harmful to both the whole union's and the country-specific welfare.

The goal of Chapter 3 is to investigate whether or not it is possible to explain the house price to GDP ratio and the house price to stock price ratio as being generally constant, deviating from its respective mean only because of shocks to productivity? Building a two-sector RBC model for residential and non-residential capital with adjustment costs to capital in both sectors, it is shown that an anticipated future shock to productivity growth in the non-residential sector leads to a large increase in house prices in the present. This property of the model is used to explain the current house price behavior in the U. S., the U. K., Japan and Germany.

In Chapter 4, the following question is posed: Can the New Keynesian Open Economy Model by Galí and Monacelli (2005b) explain "Six Major Puzzles in International Macroeconomics", as documented in Obstfeld and Rogoff (2000*b*)?
The model features a small open economy with complete markets, Calvo sticky prices and monopolistic competition. As extensions, I explore the effects of an estimated Taylor rule and additional trade costs. After translating the six puzzles into moment conditions for the model, I estimate the five most effective parameters using simulated method of moments (SMM) to fit the moment conditions implied by the data. Given the simplicity of the model, its fit is surprisingly good: among other things, the home bias puzzles can easily be replicated, the exchange rate volatility is formidably increased and the exchange rate correlation pattern is relatively close to realistic values. Trade costs are one important ingredient for this finding.

Keywords:
macroeconomics, economic policy, DSGE models, currency union, monetary-fiscal interactions, real estate economics, long run risk, New Keynesian models, small open economy, simulated method of moments

Zusammenfassung

Die vorliegende Dissertation umfasst vier Kapitel und einen Anhang. Nach einem einleitenden und einen Literaturüberblick bietenden ersten Kapitel zeigen Kapitel zwei bis vier eigenständige und voneinander unabhängige Forschungsthemen.

In Kapitel zwei wird ein Zwei-Länder Modell einer Währungsunion betrachtet, in dem die gemeinsame Zentralbank die Wohlfahrt der gesamten Währungsunion maximieren will, während die zwei fiskalpolitischen Akteure vergleichbare, aber minimal abweichende länderspezifische Verlustfunktionen zu minimieren suchen. Das Konkurrenzverhalten dieser drei Institutionen wird in sieben verschiedenen, statischen spieltheoretischen Szenarien analysiert. Beim Vergleich einer homogenen mit einer heterogenen Währungsunion lassen sich deutlich höhere Wohlfahrtsverluste relativ zum sozialen Optimum für letztere feststellen. Die Szenarien mit den geringsten Wohlfahrtsverlusten sind Kooperation aller drei Institutionen und eine Stackelberg-Führerschaft der Zentralbank. Kooperation nur zwischen den fiskalpolitischen Akteuren schadet der Wohlfahrt sowohl der Währungsunion insgesamt als auch der beiden Länder.

Kapitel drei untersucht, inwieweit das Verhältnis von Immobilienpreise zum Bruttoinlandsprodukt als langfristig konstant und nur auf Grund von Produktivitätsschocks von seinem Mittelwert abweichend angesehen werden kann. Hierzu wird ein Zwei-Sektoren RBC-Modell für den Immobiliensektor und einen Konsumgütersektor mit Kapitalanpassungskosten in beiden Sektoren erstellt. Es wird gezeigt, dass ein antizipierter, zukünftiger Schock auf das Produktivitätswachstum im Konsumgütersektor eine deutliche Erhöhung der Immobilienpreise relativ zum Bruttoinlandsprodukt zur Folge hat. Diese Eigenschaft des Modells wird verwendet, um die Immobilienpreisentwicklungen in den USA, im Vereinigten Königreich, in Japan und in Deutschland zu erklären.

In Kapitel vier wird gefragt, ob das Neukeynesianische Modell von Galí and Monacelli (2005b) die in Obstfeld and Rogoff (2000*b*) dokumentierten "sechs großen Rätsel der internationalen Makroökonomie" erklären kann. Als Erweiterungen des Modells werden die Wirkung einer geschätzten Taylorregel und zusätzliche Handelskosten untersucht. Nachdem die sechs Rätsel in Bedingungen für erste und zweite Momente übersetzt worden sind, werden fünf wesentliche Modellparameter mittels Simulated Method of Moments (SMM) geschätzt. In Anbetracht der relativen Einfachheit des Modells ist das Ergebnis erstaunlich gut: unter anderem können die empirischen Beobachtungen zur Heimatpräferenz widergegeben und die Schwankungsbreite des realen Wechselkurses deutlich erhöht werden. Handelskosten sind für dieses Ergebnis ein wesentlicher Faktor.

Schlagwörter:
Makroökonomie, Wirtschaftspolitik, Dynamische Allgemeine Gleichgewichtsmodelle, Währungsunion, Wechselwirkungen von Geld- und Fiskalpolitik, Immobilienmarkt, Langfristrisiken, Neukeynesianische Modelle, Kleine, offene Volkswirtschaft, SMM-Schätzung

Acknowledgements

Many people contributed to the completion of this thesis and I am very grateful to them. First of all, I would like to thank my supervisor Professor Harald Uhlig for inspiring me to investigate the field of macroeconomics. His enthusiasm for doing research was always stimulating and inspiring; his ideas were often an eye-opening experience for me. I would also like to thank my second examiner Professor Michael Burda for fruitful and enjoyable discussions. For some time, the atmosphere at the economics department at Humboldt-Universität zu Berlin was that of a research department, and I would like to thank all who took part in creating this atmosphere. Besides the aforementioned, I would like to name Professors Albrecht Ritschl, Bartosz Mackowiak, Monique Ebell, Alexandra Spitz-Oener, Mark Weder, Mirko Wiederholt, Michael Kvasnicka, Thorsten Vogel and all the numerous Ph.D. students I was happy to have as colleagues. Very special thanks go to my – now mostly former – colleagues at the institute: Georg Man, Almuth Scholl, Mathias Trabandt, Emanuel Mönch, Michael Fünfzig, Christian Stoltenberg, Alexander Kriwoluzky, Martin Kliem, Holger Gerhardt and Pooyan Amir Ahmadi. During the last months, I very much enjoyed the presence of Professor Günther Rehme at the institute. And of course, without the support of Susann Röthke and the institute's student assistants, things would be only half as good; as a representative for many others who helped forming this book, my special thanks go to Simon Rösel. For very careful proofreading, I would like to thank my sister Claudia Eger, Marcus Eger and my wife.

Research is a matter of cooperation and discussions. Therefore, I am very grateful for the cooperation of my coauthors Harald Uhlig and Oliver Grimm. Meeting Oliver at the very first day of my studying economics at Heidelberg was a privilege. Among the other members of the Nussloch Society for Social end Economic Sciences my special thanks go to Alexander Moutchnik for regular encouragement and for being a friend. I am also indebted to those who had an open ear for my special research papers: besides my colleagues at the institute, these were Martin Eichenbaum, Jordi Galí, Hans Gersbach, Giovanni Lombardo, and audiences at the CFS Summer School in Eltville, EcoMod conference 2004 in Paris, at the Annual Meeting of the European Economic Association 2005 in Amsterdam and 2007 in Budapest, at the Jahrestagung des Vereins für Socialpolitik 2006 in Bayreuth and 2007 in Munich, at the Spring Meeting of Young Economists 2007 in Hamburg, at the Oesterreichische Nationalbank in Vienna, as well as audiences in Bonn, Heidelberg, at the Bundesbank and the Oesterreichische Nationalbank, at WHU Koblenz and at the Macro Brown Bag seminar at Humboldt.

My research was graciously supported by the Deutsche Forschungsgemeinschaft through SFB 373 "Quantification and Simulation of Economic Processes" and SFB 649 "Economic Risk", by the research training network MAPMU and by the German Academic Exchange Service through a doctoral grant while I was visiting the University of California at Berkeley.

But most of all I would like to thank my family, and especially Monika, Jakob and Mia for their patience, their support and their love.

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1 Introduction

1.1 Objective of the Study

Macroeconomic theory has made progress during the last decades.

Dynamic stochastic general equilibrium (DSGE) models allow researchers to assess sign and size of the effect that certain changes have on a model economy. As this model economy is built up on the grounds of utility maximizing households and profit maximizing firms, one can infer the reaction of each agent to these changes, be they stochastic disturbances or policy switches.

Game theory, applied to macroeconomics, allows researchers to find out - among other things – how a group of policy authorities interacts and how this affects macroeconomic outcomes. Questions of coordination, timing of decisions and policy rules can be answered in this context.

The microfounded nature of modern macroeconomic models allows researchers to calculate welfare from a quadratic approximation of the household utility. Thus, welfare effects of inflation and output variability can be examined. Given a set of assumptions, within a DSGE model researchers can calculate the welfare maximum and, if an economic policy authority is included in the model, derive implications for economic policy.

Economic policy has a direct influence on some important macroeconomic variables: taxes, subsidies and government spending on the fiscal side, the short term interest rate, money supply and refinancing conditions on the monetary side, and it has the power to influence all economic agents.

As economic policy decisions are made for a maximization purpose, information on the functioning of the economy has a positive value, and macroeconomic theory has the potential to provide this information. Hence, economic policy has and should have an interest in the developments of macroeconomic theory.

The goal of this maximization does not matter in general, it is itself the object of economic theory. Most probably it is maximization of utility, something between the utility of those who decide about policy and the utility of all agents, i.e., welfare.

Macroeconomic theory claims to know something as promising as the “optimal policy” that maximizes welfare.

It is thus well prepared to give advice. This advice should take into account both the limitations of theory and the constraints of policy, and it should consider the consequences that came about with these.

In this dissertation, I provide three examples of questions economic policy might have. Three models are used that apply current macroeconomic theory to address these questions. I give answers to the questions and analyze the scope of applicability of model results to reality.

The first question to be addressed is: What are the welfare effects if there are conflicts of interest between different policy authorities in a heterogeneous monetary union? In light of the ongoing enlargement of the European Economic and Monetary Union (EMU), the economic differences between old member countries and the accessing middle and Eastern European countries come more into focus. The resulting heterogeneities between the member countries increase the probability that the respective fiscal policy authorities differ in their interests. At the same time, the common monetary policy authority continues to look at the monetary union as a whole.

In addressing the above question in joint work with Oliver Grimm, we make use of two of the three aforementioned progresses in research, namely game theory applied to macroeconomics and welfare derivation. We develop a model of two regions that form a monetary union, but still afford regional fiscal policies. All three policy authorities, one monetary and two fiscal, use their policy instruments to influence output and inflation in their respective desired direction. However, there are two crucial differences between the policy authorities. First, while the monetary authority considers aggregate union-wide variables, each fiscal authority considers merely the respective regional variables. Second, the authorities do not agree on the maximization problem. Specifically, we assume that monetary policy maximizes union-wide welfare, but fiscal policies deviate from maximizing the respective regional welfare. The result of this setting is a game between the three policy authorities, and the outcomes of this game depend on the timing of action and the degree of cooperation between the authorities. We explore what happens to the results when we deviate from the assumption of symmetric regions. Beginning with the size of the regions, we investigate the effects of heterogeneities in a set of model parameters on the economic outcomes in the regions and the union, keeping a focus on welfare.

The second exemplary question that economic policy is eager to have answered belongs to the field of real estate. House price movements in the United States, the United Kingdom and recently in Spain have found increasing attention by domestic politicians, who wonder how to react to them and whether or not specific action is demanded. The research project here, carried out jointly with Harald Uhlig, was inspired by a discussion forum on the recent developments in the real estate market at the German Ministry of Economics and Technology. To find out to what extent current house price developments can be explained by economic theory, we use a standard DSGE model that incorporates the real estate market. We take this as our specific research question: Is it possible to explain the house price to GDP ratio and the house price to stock price ratio as being generally constant, deviating from its respective mean only because of shocks to productivity? If so, economic policy may be little concerned about current price fluctuations, but should strongly focus on the improvement of long-term growth conditions. The DSGE model we build has two sectors, one for consumption goods and one for real estate. Real estate is produced using capital and finite land; it provides housing services that enter the household utility function. We specifically focus on trend productivity and productivity growth to examine to what extent expected future productivity increases can explain current house price movements.

The third example of policy questions belongs to the field of monetary economics. Personal discussions both at the Deutsche Bundesbank and at the Oesterreichische Nationalbank confirmed that there is an interest in building a small scale state-of-the-art New Keynesian DSGE model to be used for the conduct of monetary policy. The New Keynesian paradigm is currently dominating the field of monetary economics; it incorporates the Keynesian assumption of sticky prices (and/or wages) into the DSGE framework with rational expectations that was earlier used mainly by Real Business Cycle theorists. While the canonical New Keynesian model for a closed economy is well documented, e. g. in Clarida, Galí, and Gertler (1999) or Woodford (2003), the literature is not yet clear about a definitive New Keynesian open economy model. For the case of a small open economy, the paper by Galí and Monacelli (2005b) is a potential candidate.

Just like the canonical closed economy model, the Galí and Monacelli (2005b) model is of small to medium scale and reasonably simple. The restricted scale of the model is often favored by economists in (and out of) central banks, because with a small-scale model it is easier to get an intuition of what is happening and which are the driving forces. The drawback of this is the lack of sufficient modeling features to replicate a multitude of stylized facts that have been derived from the data. This is the old debate between simplicity of a model and its proximity to reality. If one decides in favor of simplicity, how much does it cost in terms of deviation from reality? If a model terribly fails to replicate a certain list of stylized facts, these costs are high, and so is the probability that relying on this model alone will be misleading. In this research project, I test the Galí and Monacelli (2005b) model for its closeness to reality with respect to six stylized facts in international macroeconomics, as documented in Obstfeld and Rogoff (2001). This is done by choosing a set of model parameters to minimize the distance between certain moments of the model and those of the data. To simplify the task, I first estimate a Taylor-type rule for the small open economy's monetary policy instead of using the strict targeting rules that are used in the original paper. In a second step, I test the hypothesis of Obstfeld and Rogoff (2000*b*) whether trade costs help to get the model closer to the data.

In the next section, I review the literature that is relevant to each specific research problem.

1.2 Literature Review

1.2.1 Policy Interactions in a Heterogeneous Monetary Union

Kydland and Prescott (1977) were the first to explore the drawbacks of discretionary policy; Barro and Gordon (1983) followed on that topic. Both papers focused on one policy authority, namely the central bank, surprising the agents by sudden inflation which creates short-term benefits. These two papers form the basis of our research. However, in two respects they differ strongly from our approach. First, they lack a maximizing fiscal policy authority and thus cannot analyze the resulting interactions between the two authorities. Second, the peculiarities of a monetary union are not explored in these models.

The first paper that explicitly examines the interactions between maximizing monetary and fiscal authorities in a closed economy and thus overcomes the first major difference to our approach is Alesina and Tabellini (1987). In this paper, both monetary and fiscal policies maximize a function in inflation, output and government expenditures with respect to inflation and taxes, respectively. A similar model is presented in Debelle and Fischer (1994). Here, the timing of action plays a prominent role: Nash and Stackelberg games are considered.

The analysis of monetary and fiscal policies within the framework of a monetary union appears first in a series of papers in the nineties. Banerjee (1997) is perhaps closest to Barro and Gordon (1983). He includes the monetary union case into this framework, but his emphasis is on the issue of rules versus discretion. Sibert (1992), Levine and Brociner (1994) and Beetsma and Bovenberg (1998) all consider monetary and fiscal policies in a common currency area. Their fiscal policies are to provide public goods and thus differ from our focus on inflation and output stabilization. They conclude that discretionary policies lead to a bigger than optimal amount of public goods and too high an inflation rate.

In the context of the analysis of fiscal policy in a monetary union, two topics have received considerable attention. One is the desirability of fiscal constraints like the Stability and Growth Pact for the EMU. Chari and Kehoe (1998) and Dornbusch (1997) argue against fiscal constraints in a case where the common central bank can commit to its policies. Beetsma and Bovenberg (1998) and Beetsma and Uhlig (1999) take the opposite view; their models focus on the short-sightedness of fiscal policies with respect to the external effects of a singular fiscal authority's action on union-wide inflation. Hence, they come to the conclusion that fiscal constraints are improving welfare.

The second important topic is that of the desirability of fiscal coordination in a monetary union. Chari and Kehoe (1998) and Huber (1994) recommend coordination from a welfare perspective, as it reduces excessive debt-taking. The opposite position is taken by Beetsma and Bovenberg (1998). Their reasoning is based on a model in which fiscal policy takes action ahead of monetary policy. There, short-sighted fiscal policy gets more effective once it is coordinated, to the bad of welfare.

With his Alfred Marshall Lecture at the Annual Meeting of the European Economic Association in the year 2000, Avinash Dixit brought the issue of policy interactions in a monetary union to a wide scientific public. Some of the topics sketched there have been elaborated in research papers that form the starting point of Chapter 2 of this dissertation. Dixit and Lambertini (2001) study an n-country monetary union with maximizing fiscal and monetary policies. All $n+1$ policy authorities minimize a loss function in union-wide inflation and country-wide output, except for the central bank, which considers also union-wide output. Target levels for output and inflation as well as the relative weight of output in the loss function may vary between the policy authorities, so there is room for conflicts. In a Barro and Gordon (1983) like manner, each country-wide output is influenced by surprise inflation and all fiscal policies, whereas overall inflation is linearly dependent on the actions of all $n+1$ policy authorities. The authors analyze (a) a simultaneous game of all authorities against each other, i.e. a Nash equilibrium, (b) the case where monetary policy acts first, i.e. monetary leadership in a Stackelberg game, (c) fiscal leadership and (d) monetary commitment. With the assumption of a conservative central bank that has lower output and inflation targets and a weight on output in the loss function that is not bigger than that of the fiscal authorities, they conclude that (i) the Nash outcome is always suboptimal, (ii) the leadership scenarios might be less suboptimal and (iii) monetary commitment proves of no additional value in a world with fiscal discretion, as the outcome is the same as under monetary leadership. The last result is also shown in Dixit and Lambertini (2003*a*) for the case of a closed economy. This paper includes also the cases of fiscal and joint commitment, and of nonstrategically chosen fiscal policy. Next to these, the paper gives a simulation of results, adds a microfoundation of the model equations and calculates welfare, as derived from a second-order approximation of household utility. Lambertini (2006a) is a digression from Dixit and Lambertini (2003*a*) with respect to the goal of fiscal policy: instead of providing a production subsidy, here fiscal policy collects taxes for utility-providing government spending. Assuming fiscal policy to maximize social welfare, she is able to rank equilibria accordingly. Dixit and Lambertini (2003*b*) adds the microfoundation of Dixit and Lambertini (2003*a*) to the monetary union case. It investigates in greater detail a situation in which all policy authorities agree upon the target values for output and inflation. The authors show that in such a situation of symbiosis, the policy targets can be achieved no matter what the order of action, how the weights of the objectives, whether or not monetary commitment is feasible, and whether or not fiscal authorities cooperate. The case of identical target values is also analyzed in Cooper and Kempf (2000). Assuming that the budget constraints are shared, the common central bank case is compared to the one with national central banks. Lambertini (2004) adds a derivation of social welfare and elaborates on the issues of fiscal cooperation. It turns out that fiscal cooperation typically worsens the results in the Nash equilibrium. In contrast to this, Lombardo and Sutherland (2004) come to the conclusion that fiscal cooperation leads to small, but not negligible welfare gains. This is found in a two-country model where equilibrium government consumption is positive, public goods provide utility and supply shocks are not perfectly negatively correlated. Uhlig (2002) gives some remarks on the fiscal leadership equilibrium in an stylized model of an n country monetary union. Among other things, he shows that there is a free-riding issue for fiscal policies which results in a higher interest rate. This issue might be resolved if all fiscal authorities cooperate. EMU‘s Stability and Growth Pact might be seen as a step towards resolving this free-riding issue.

The papers discussed so far all share one modeling feature: the time dimension is fairly restricted. The static models used know only two situations, pre- and post-action. Nonetheless, the timing of the actions itself may be quite delicate and elaborated.

The issue of policy interactions in a monetary union is also dealt with in dynamic models. Here, the literature can be divided into two strands. One strand uses highly stylized models with ad hoc macroeconomic equations and without a proper derivation of the relevant policy maximization problems. But these models typically include dynamic games, in particular open-loop Nash and cooperative equilibria. For example, Engwerda, van Aarle, and Plasmans (2005) assume monetary policy to passively fix the nominal interest rate and analyze Keynesian fiscal policies. They especially focus on the effects of fiscal constraints like the Stability and Growth Pact and rate them possibly harmful to the economy. They also evaluate the effects of fiscal transfer mechanisms between the two countries of the model. van Aarle, Engwarda, and Plasmans (2002) analyze dynamic games between three policy authorities, one monetary and two fiscal. They differentiate the Nash equilibrium from full cooperation and three variants of partial cooperation. Varying the degree of symmetry between the two countries in different respects (stabilization preferences, monetary policy transmission, bargaining power in a coalition, sensitive to intra-union competitiveness), they come to the following conclusions: First, it pays off for the fiscal authorities to cooperate, at least if the countries are sufficiently symmetric. Second, the more asymmetric the countries are, the more likely is the Nash equilibrium. Monetary policy does not profit from cooperation. Third, cooperation between the fiscal authorities results in higher losses for the central bank. Lastly, cooperation between the monetary and one fiscal authority leads to suboptimal results.

The paper of Canzoneri, Cumby, and Diba (2005) is located somewhat in between the two strands of literature. These authors construct a partial equilibrium New Keynesian model to look at the effects of inflation targeting monetary policy on different countries within a monetary union.

The second strand uses the New Keynesian DSGE framework with second order welfare approximation. However, these models typically incorporate less features of dynamic game theory. Instead, they focus more on optimal policies in comparison to simple policy rules.

Within the context of a closed economy, Schmitt-Grohé and Uribe have analyzed optimal policies in a series of papers. In Schmitt-Grohé and Uribe (2004), e.g., they solve the Ramsey problem of a benevolent joint policymaker for monetary as well as fiscal policy. They compare this outcome to the outcomes in case either monetary or fiscal policy follows some simple rule, while the other remains to solve the now more constrained Ramsey problem. They find that the simple rules assumed do not harm welfare substantially. Similarly, Benigno and Woodford (2003) assume jointly optimal monetary and fiscal policies. They derive optimal policy responses to shocks and optimal targeting rules. In solving a Ramsey problem, the authors of the two previously mentioned papers assume that the policy authorities have commitment power. In a similar model Adam and Billi (2006) analyze the case in which one or even both authorities do not have access to a commitment device. They work out that the welfare loss due to fiscal discretion is rather low. However, the welfare loss of monetary discretion turns out to be significant, yet can be overcome by making the central bank sufficiently conservative with respect to the inflation target or the inflation weight in the central bank‘s loss function.

The case of optimal monetary policy in a currency union is dealt with in Benigno (2004). In this beautifully designed two-country model, an assumption is made regarding fiscal policy that can be seen in a multitude of New Open Economy Macroeconomics models: fiscal policy is used to offset the distortion of monopolistic competition. Hence, fiscal policy is time invariant and not strategic by any means.

Ferrero (2005) extends the work of Benigno and Woodford (2003) to a monetary union. Assuming that both fiscal and monetary policy care about union-wide welfare, he solves the optimal policy problem and compares the results to simple rules, strict CPI inflation targeting on the monetary side and constant debt on the fiscal side. Ferrero finds that the welfare loss of a constant debt rule is much higher than the welfare loss due to strict CPI inflation targeting. In a similar way, Galí and Monacelli (2005b) solve the social planner problem for a monetary union that consists of infinitely many and infinitesimally small open economies. Fiscal policies are modeled by government spending, financed by lump sum taxes. The authors find that in response to asymmetric technology shocks fiscal policies should optimally increase the provision of public goods. In a similar way, Beetsma and Jensen (2004) analyze benevolent optimal monetary and fiscal policies in a monetary union. Fiscal policy in this paper also means providing public goods, financed by lump sum taxes or deficit. The authors also provide a comparison between commitment and discretion of all authorities and find sizeable gains of commitment. Lambertini (2006b) starts off with the assumption that monetary policy follows a Taylor rule, whereas the two fiscal authorities cooperate to solve the Ramsey problem for the monetary union, as it is also done in Schmitt-Grohé and Uribe (2004). Fiscal policy means choosing a labor tax rate and a deficit to finance an exogenously given stream of government spending. Lambertini then implements the Stability and Growth Pact as an additional constraint to the Ramsey problem. As her point of departure is the optimal fiscal policy, this additional constraint comes with a cost in terms of welfare. Fortunately, this cost is small, as the constraint is rarely binding for the assumed optimal fiscal policy.

The objective of Chapter 2 of this dissertation is to analyze the interactions between monetary and fiscal policies in a heterogeneous monetary union. We take the view that commitment is not feasible and restrict ourselves to discretionary policies. Hence, we have decided against a dynamic model setting. Obviously, the dynamic approach to macroeconomic policies in a monetary union has its pros. Just to mention one point, deficits and debt are of utmost importance to fiscal policy, and even more so in a monetary union with its spill-over effects and the danger of free-riding. A static model cannot account for this properly. On the other hand, the games that may be played between different policy authorities in a monetary union have not yet been analyzed in depth in a dynamic setting. If one assumes commitment to be infeasible, the DSGE literature becomes quiet on the topic. Only the strand of literature with ad hoc macroeconomic equations provides some insights here. Thus, van Aarle et al. (2002) is a dynamic counterpart to our study. In the class of static models, our point of departure are the models of Dixit and Lambertini (2001, 2003*a*,) all of which we can replicate in our model. However, we deviate from these in many respects to focus on heterogeneities in a monetary union. First, instead of n countries our monetary union consists of two countries of possibly different size. Thereby we are able to analyze differences within a monetary union in the way of Benigno (2004) or Ferrero (2005) for EMU countries. Second, we take the view fiscal policies care about inflation on the country level, not union-wide. Among other things, this leads to terms of trade entering the model equations. Third, we investigate the effects of heterogeneities in the model parameters on output, inflation and welfare, both for the union and each region separately. In doing so, we can hint on the effects of heterogeneities and perhaps also on the desirability of a fast expansion of the European Monetary Union.

1.2.2 The Real Estate Market from a Macroeconomic Perspective

In general, macroeconomists are not too much concerned about the real estate market, taking it as just one of many sub-markets that should function as good as others. But from time to time and from country to country, this market shows some peculiarities that attract more than usual attention among economists. The last half decade has been such a time, and the U.S. and the U.K. – to name the biggest out of a longer list – have been such countries.

Once economists take a special look at the real estate market, they try to single out features that make this market special – hence worth considering – and allow for explanation for the observed peculiarities.[1] In the following, I review some of the leading theories that investigate and try to explain peculiarities in the real estate market. While my focus is on the explanation of house price movements, I also look at related works.

One line of research focuses on heterogeneous agents and how they are affected by house prices. Iacoviello (2005) and Campbell and Cocco (2005) both take house prices as given and ask how these affect households, especially their consumption paths. Iacoviello (2005) relates a borrowing constraint to the borrower's home value. Once house prices fall, borrowing becomes restricted at least for some households, and consumption behavior changes as a result of this. Campbell and Cocco (2005) show different effects of house price movements on consumption for different age groups and build a heterogeneous agent model with borrowing constraints to replicate the stylized facts. The paper of Piazzesi and Schneider (2008) combines heterogeneous agents with another issue, so it shall be discussed below.

The role of credit market imperfections is also present in Stein (1995), who assumes that heterogeneous households face a minimum down payment condition when buying a house. With this, the author addresses the correlation of house prices and trading volume. Ortalo-Magné and Rady (2006) build on the previous paper and have a closer look at households' decisions to buy a house. Assuming heterogeneous agents with respect to income and utility of high quality housing, as well as mortgage restrictions, the authors show how changes to these assumptions induce strong house price movements and even overshooting. The upturn in house prices at the beginning of this decade can thus be rationalized by lighter mortgage restrictions.

In another line of research, the effect of inflation on house prices is analyzed in a se-

[1] A noticeable exception is Poterba (1991), who tries to explain the nominal U.S. house price decline at the end of the eighties by treating housing just like any other asset.

ries of papers by Poterba (1984, 1991, 1992). The channel of inflation to house prices is through an effective tax subsidy to owner occupied housing. A more recent approach is taken by Piazzesi and Schneider (2008), who build a heterogeneous agent model in which next to the tax channel heterogeneous inflation expectations increase the volume of credit and thus the price of the collateral. Brunnermeier and Julliard (2008) analyzes the effect of inflation through money illusion: if agents assume nominal and real interest rates move one-to-one, they wrongly identify a decrease in the inflation rate as lower real interest rates. This makes them more willing to accept a mortgage contract, so house prices increase. With this theory, they explain the house price increase at the turn of the millennium by decreases in inflation rates.

Piazzesi, Schneider, and Tuzel (2007) see housing as an asset that pays of a consumption good, namely housing services. These housing services provide utility that can not be separated from consumption utility. Their goal is to show effects of housing on asset prices and not to explain house price movements itself. With their endowment economy with nonseparable utility of consumption and housing services they show that the housing share in total consumption is useful for forecasting excess returns on stocks.

Davis and Heathcote (2005) single out three distinctive features of housing or residential investment: its production makes use of different inputs shares, it is of different use and it has different business cycle dynamics, all to nonresidential investment. They model the production side as intermediate sectors for construction, manufacturing and services and final sectors for consumption/investment and residential structures. With this model at hand, they replicate the relatively high volatility of residential investment and its procyclicality, though not the lead-lag pattern of the data. Finally, there have been attempts to explain house price movements by search and matching models. Wheaton (1990) is a key reference here.[2]

Chapter three of this dissertation is an attempt to explain house price movements relative to the gross domestic product. Without relying on heterogeneous agents, credit market imperfections or any nominal frictions, house prices are influenced only by productivity shocks, both current and anticipated future shocks. Under the label of “long-run risk”, the latter type of shocks has recently attracted attention, see Bansal and Yaron (2004) and Hansen, Heaton, and Li (2005), though both do not look at the real estate.

[2] See Ortalo-Magné and Rady (2006) for further references.

1.2.3 Confronting Small-Scale Open Economy Models to the Data

Since the path-breaking work of Obstfeld and Rogoff (1995), DSGE models with price rigidities are more and more used for open economy monetary policy analysis. This paper may be seen as the beginning of the so-called New Keynesian or New Neoclassical Synthesis approach for the open economy. Nowadays, it has become the standard in the field. The advances since then are well documented in the literature, first of all Lane (2001). Textbook treatments of open economy models are are given in Obstfeld and Rogoff (1996, ch. 10) and, more recently, Galí (2008, ch. 7). From a theoretical point of view, open economy macroeconomic models are typically divided in models with two (or more) countries of comparable size and models where one country is so small that it is affected by, but itself cannot affect the rest of the world: the small open economy setting. An early and particularly well known example of the latter is the model of "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", a paper by Jordi Galí and Tommaso Monacelli, first circulated in 1999 and published 2005 in the Review of Economic Studies.

With only one source of shocks (technology) and only two frictions (monopolistic competition, sticky prices), the model is small of scale and fairly simple. Logically, it cannot be an adequate model for every question in open economy macroeconomics. But as researchers are always tempted to start off with simple and standard models to answer their questions, the scope of applicability deserves some attention. A good way to evaluate this scope of applicability is to check whether the model can replicate some stylized facts in macroeconomics. The particular stylized facts that are used in Chapter 4 of this dissertation as a benchmark are summarized in "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?", a paper by Maurice Obstfeld and Kenneth Rogoff in the NBER Macro Annual 2000. In this literature review, I will not discuss whether these "six puzzles'" represent stylized facts. And I will also not discuss whether this set of stylized facts is an appropriate one - this is always dependent on the actual research question one wishes to focus on. Of course, any other set of stylized facts might have been chosen here as well.

Instead, I will focus on methods to put current small to medium scale open economy DSGE models to the data. In describing and distinguishing these methods, I rely on the textbook by Canova (2007). A lucid and much more detailed description of the methods can be found there. There are two main avenues researchers have taken to put open economy macro models to the data. One avenue is that of calibration, as proposed by Kydland and Prescott (1982, 1991). The other avenue is estimation, and this avenue splits into four roads: (i) Vector autoregression (VAR), as first presented in Sims (1980). The validation of a theoretical model with the help of a structural VAR is connected to the work of Christiano, Eichenbaum, and Evans (2001). (ii) Maximum likelihood (ML), a method used already for a long time. It starts from the assumption that the model is a correct representation of the underlying data generating process, while only the parameters have to be chosen so as to maximize the likelihood function. (iii) Generalized method of moments (GMM) starts from the same assumption, but uses less information about the model. The pioneering researchers are Hansen and Singleton (1982). Comparable to this method are simulation estimators. (iv) Bayesian methods have only recently been applied to DSGE models, following the lead of Geweke (1999) and Schorfheide (2000).

In the following, I briefly describe each method and review some of the contributions that have been made to the literature on confronting small scale open economy DSGE models to the data.

Calibration

The calibration procedure can be described as follows:[3] (1) Choose an economic question to be addressed. (2) Select a model design which bears some relevance to the question asked. (3) Choose functional forms for the primitives of the model and find a solution for the endogenous variables in terms of the exogenous ones and of the parameters. (4) Select parameters and convenient specifications for the exogenous processes and simulate paths for the endogenous variables. (5) Evaluate the quality of the model by comparing its outcomes to a set of "stylized facts" of the actual data. (6) Propose an answer to the question, characterize the uncertainty surrounding the answer and do policy analyses if required.

Backus, Kehoe, and Kydland (1995) ask whether "a two-country real business cycle model can account simultaneously for domestic and international aspects of business cycles."[4] Setting up their model, simulating it and comparing the thus obtained second moments to those obtained from data for the U.S. and some European countries, they find two major discrepancies. As their attempts to solve them do not succeed, they name them the "quantity anomaly" and the "price anomaly". The first refers to the observation that relative to output, consumption is more internationally correlated in theory, but less in the data. The second refers to the terms of trade. In the data, they

[3] Taken from Canova (2007, ch. 7).
[4] Backus, Kehoe, and Kydland (1991)

are very volatile and persistent, while the model can replicate at most 20 percent of the volatility of the terms of trade. Both anomalies are at the core of stylized facts I international macroeconomics that models have a hard time to replicate. Under the headings consumption correlation puzzle and exchange rate volatility puzzle, they also show up in Obstfeld and Rogoff (2001) six major puzzles.

Kollmann (2001) focuses on the second anomaly. He uses the observation of Backus et al. (1995) as a starting point and builds a small open economy DSGE model with nominal rigidities in the line of Calvo (1983) to see whether this "would allow to capture simultaneously the high volatility of exchange rates and the other key macroeconomic facts considered"[5] His model can explain about 50 percent of the volatility of the real and nominal exchange rate.

Chari, Kehoe, and McGrattan (2002) pursue an effort in the same direction: "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?" Their baseline model with prices set a year in advance, high risk aversion and linearly separable utility of leisure is able to match the volatility most of the persistence of the real exchange rate, but at the cost of a new anomaly: They find that their model implies a high and positive correlation between real exchange rates and the ratio of consumptions across countries, whereas the data shows no clear correlation. Among the means considered to circumvent the anomalies, Chari et al. (2002) consider different parameter values, different shocks and different model assumptions regarding preferences and monetary policy. A noteworthy detail of their paper is the calibration of the interest elasticity. The value for that is obtained from a single equation OLS regression on the model‘s money demand equation, thus opening a door for estimation in a calibration paper.

This method is silent on the question of how to choose parameters to optimize the model fit. It often seems to be a process of guess and verify. In a more dimensional problem, this is perhaps not sufficient for every researcher.

Vector Autoregression

The vector autoregression (VAR) methodology plays a prominent role for the evaluation of macroeconomic models since its beginning with Sims (1980).Evaluation is done by comparing statistics of the DSGE model with those of a VAR model. If the further are within certain probability bands of the latter, this is seen as confirmation of the DSGE model's quality. A summary of the algorithm according to Canova (2007, Section 4.7) is

[5] Kollmann (2001, p.260)

given as follows: (1) Find qualitative, robust implications of a class of models. (2) Use these implications to identify shocks in the actual data. (3) Evaluate the model qualitatively, e.g. by the sign or shape of impulse responses to shocks. (4) If there is more than one model at hand, validate qualitatively across models. (5) If needed, compare model and data quantitatively. (6) If needed, repeat steps (2) to (5) for other implications of the model. (7) Proceed to policy analyses, or alter the model in case of mismatch. In the literature, VAR models have been used to find out how economies react to shocks. The results found using VARs are then compared to the respective responses of a DSGE model. Clarida, Galí, and of Dallas (1994) as well as Eichenbaum and Evans (1995) both look at the VAR evidence on the effect of monetary shocks on the real exchange rate. They are able to find the same qualitative responses in their sticky price models. Betts and Devereux (1996) also include the trade balance in their model and show that their DSGE model with pricing to market improves on both the quantity anomaly and the price anomaly. Another emphasis is put on the question whether monetary shocks affect the current account. Lane (1998); Prasad (1999) and Lee and Chinn (1998) all find a slight improvement in the current account after a positive monetary shock, though their identification scheme of a monetary shock differs.

Ghironi, Iscan, and Rebucci (2003) uses the two-country model of Ghironi (2000) to see whether differing discount factors and steady-state productivity levels help to explain net foreign asset holdings and the quantity anomaly. After making identification assumptions for the assumed productivity shocks, they estimate a VAR and compare the responses of the model to those of the VAR. While asymmetries of the current account can be well reproduced, the low consumption correlation of the data is not completely replicated in the model.

Though the use of VARs is widespread, there are nonetheless certain dangers. The perhaps most important one is model misspecification, either because of omitted variables or because shocks are misaggregated.[6]

Maximum Likelihood

The estimation of a model with maximum likelihood (ML) is grounded on the strong assumption that the model is the true data generating process. Estimating the structural parameters using ML requires (1) writing the model in state space form and (2) applying the Kalman filter to obtain optimal recursive estimates of the unobserved state variables and minimum mean square error forecasts of the endogenous vari-

[6] See Canova (2007, Section 4.6) for examples.

ables.[7] A first attempt to apply ML to an open economy model is Ghironi (2000). He builds a deterministic two-country model with population growth and price rigidities. He estimates his model in two ways: First by single equation OLS, then by full information maximum likelihood estimation of each the supply and the demand side of the economy using data for Canada and the U.S.

Bergin (2003) uses the small open economy model of Kollmann (2001) to estimate its log-linear approximation with the maximum likelihood procedure of Leeper and Sims (1994). For the estimation process, he uses the seven time series current account, nominal exchange rate, domestic and foreign price indices, output, money supply and world interest rate. As small open economy he uses data of Canada, Australia and the U.K. He finds that the model does a reasonably good job with respect to prices and output, whereas exchange rate movements are badly matched. Bergin compares the structural model estimates with VAR (1) estimates and with estimates of an unrestricted model in which the interrelations of parameters over the model equations are not considered. He finds that the model outperforms the VAR for all three countries and outperforms the unrestricted model for all countries analyzed except for the U.K. Furthermore, he investigates whether Calvo (1983) price and/or wage stickiness is a reasonable feature and whether pricing to market or producer currency pricing is the better assumption. His estimates vote strongly in favor of nominal rigidities. Out of them, price stickiness is more often supported than wage stickiness. Regarding the currency denomination of exports, producer currency pricing fares a bit worse than pricing to market, as the model outperforms the unrestricted model only in the case of Australia.

Benigno (2004) is a similar exercise in a two-country model with a rich set of frictions. The econometric method is the same as in Bergin (2003), but data is now for the U.S. versus the remaining G7 countries. Among his findings are that a shock to the uncovered interest parity condition has high explanatory power for the current account, but not so for the exchange rate, which is more dependent on monetary policy shocks. Overall, the model fits the data “reasonably well”.

A ML estimation of a typical real business cycle (RBC) DSGE model, augmented with a VAR specification of the error terms, is provided in Ireland (2004).[8] While confirming some of the standard findings for RBC models, he disappointingly finds that the hypothesis of stable structural parameters of the sample period is rejected.

[7] See algorithm 6.1 in Canova (2007) for a description of the Kalman filter.

[8] Therein the interested reader will find also some good sources on other papers.

Among the most prominent issues of ML estimation are[9] unobservable variables in the state vector, the number of series used in the estimation, the quality of the estimates, given the assumption that the model is the true data generating process, and finally the identifiability of parameters.

Generalized Method of Moments and Simulated Method of Moments

A generalized method of moment (GMM) estimator is chosen as to minimize the weighted squared distances, mostly between sample and population orthogonality conditions. In contrast to maximum likelihood, only limited information of the model is used. Often, only single equations are estimated. Examples in the open economy literature are Imrohoroglu (1994) for an estimation of currency substitution and Clarida, Galí, and Gertler (1998) for monetary policy rules. The latter estimate Taylor type rules for the U.S., Japan and the big four Western European countries. Among others, they find that exchange rates and foreign interest rates have negligible effects on monetary policies. A big issue for GMM estimation are the properties of the estimator when obtained with a small sample. According to experiments, asymptotic theory applies for sample sizes above 300 periods, which for quarterly data is more than 75 years.[10] Another issue is the optimal choice and amount of instruments to be used: more instruments improve asymptotic efficiency, but also increase small sample bias.[11]

Bayesian estimation

When distinguishing Bayesian from classical econometrics, Sims (2002) uses the example of how to use a testing device on an assembly line. “While all these classical [econometric] procedures [of using the device] are associated with probability statements about how the procedures behave across repeated measurements, independent of the true state being measured, Bayesian inference aims instead at making probability statements about the true state of the world given a particular measurement or set of measurements.”[12] This example highlight two major aspects in which Bayesian econometrics differs from classical econometrics, objectivity and randomness. Lancaster (2004) has given lucid explanations to both in his textbook, which shall be cited.

Bayesian inference is not “objective.” Some people, believing that sci-

[9] This list is collected from Canova (2007, Section 6.4).
[10] Canova (2007, p. 196).
[11] Ibid.
[12] Sims (2002, pp. 2f.).

> ence must be objective and its methods objectively justifiable, find this a devastating criticism. Whatever the merit of this position it does not seem to be the way applied econometrics is practiced. The typical seminar in our subject appears to be an exercise in persuasion in which the speaker announces her beliefs in the form of a model containing and accompanied by a set of assumptions, these being additional (tentative) beliefs. She attempts to persuade her audience of the reasonableness of these beliefs by showing that some, at least, embody "rational" behavior by the agents she is discussing and promising that other beliefs will, in fact, be shown by evidence to be not inconsistent with the data. She then presents her results and shows how some of her beliefs seem to be true and others false and in need of change. The entire process appears to be subjective and personal. All that a Bayesian can contribute to this is to ensure that the way in which she revises her beliefs conforms to the laws of probability and, in particular, uses Bayes' theorem.[13]
>
> In the traditional literature we often find phrases such as "x is random" or "we shall treat x as random" or even "we shall treat x as fixed, i.e. as not random" where "random" means that the object in question will be assigned a probability distribution. In the Bayesian approach all objects appearing in a model are assigned probability distributions and are random in this sense. The only distinction between objects is whether they will become known for sure when the data are in, in which case they are data(1); or whether they will not become known for sure, in which case they are parameters. Generally, the words "random" and "fixed" do not figure in a Bayesian analysis and should be avoided.[14]

This said, Lancaster describes the Bayesian algorithm:

> 1. Formulate your economic model as a collection of probability distributions conditional on different values for a model parameter $\theta \in \Theta$.
> 2. Organize your beliefs about θ into a (prior) probability distribution over Θ.
> 3. Collect the data and insert them into the family of distributions given in step 1.

[13] Lancaster (2004, p. 8).
[14] Lancaster (2004, p. 9).

4. Use Bayes' theorem to calculate your new beliefs about θ.
5. Criticize your model.[15]

In recent years, Bayesian methods have attracted considerable attention for the purpose of estimation of structural models. The landmark paper of Smets and Wouters (2003) was a breakthrough for closed economy models of medium scale, and the first open economy versions of thus estimated medium scale models started to circulate. Examples are De Walque and Wouters (2004) and Adolfson, Laséen, Lindé, and Villani (2005). For the small scale models that are of interest here, Lubik and Schorfheide (2007) is the first and prominent example using Bayesian techniques. Their model is a simplified version of the Galí and Monacelli (2005b) model that plays a prominent role in Chapter 4 of this dissertation. Their research question is very much comparable to the one of Clarida, Galí, and Gertler (1998): do exchange rates play a role in monetary policies of certain small open economies? Taking data from Australia, Canada, New Zealand and the U.K., the authors find the nominal U.S. Dollar exchange rate to be of importance for Canada and the U.K., but not important for the remaining two countries. In a follow up paper, Lubik and Schorfheide (2005) apply Bayesian techniques to a small-scale two country model, arguing that for theoretical questions two country models are of higher importance than small open economy model, so that there should be a two country model estimated with Bayesian techniques as well. In the words of Lubik and Schorfheide (2005), "the Bayesian framework is rich enough to cope with misspecification [as a result of the small scale of the model] and identification problems [that arise in large-scale models with many shocks]". Having said this, the authors nonetheless find that having less restrictive priors leads to a better model fit, which indicates that either the assumptions about the priors or the model have deficiencies. Furthermore, they conclude that new open economy models "are still very far away from offering a satisfactory explanation for exchange rate dynamics."[16] Though the Bayesian approach currently seems to be relatively attractive to researchers, here, too, are critics. In particular, the procedure to choose prior distributions of certain tightness sometimes looks as if it is result driven instead of knowledge driven. Another critique is regarding the complexity of this method, which makes it easy to keep errors unseen, just like it is much easier for a pianist to have a wrong key unnoticed when playing Shostakovich instead of Brahms. But of course, this is more of an observation instead of a critique: both Shostakovich and Brahms have utmost beauty in their works.

[15]Lancaster (2004, p. 9).
[16]Lubik and Schorfheide (2005, p. 30).

The objective of Chapter 4 of this dissertation is to confront a specific, small scale, small open economy model to specific observations from the data. When deciding about the appropriate econometric procedure, the following considerations were made. As a starting point, I have calibrated a set of parameters that seem to have agreed upon values or that have proven to be of minor importance to my research question. As Hansen and Heckman (1996) put it, "calibration is selecting parameter estimates with an implicit loss function."[17] This procedure is seen throughout the literature, though it seems to be that researchers are not particular proud about calibrated parameters in an otherwise estimated model. As the observations consist only of first and second moments, I do not need the full apparatus of a VAR. Furthermore, as I have only one source of shocks, misaggregation would be a possible issue here. As the model considered is highly stylized, I refrain from assuming that the model is the true data generating process. So maximum likelihood methods seem inappropriate. Generalized method of moments is in danger of leading to biased estimates because of the small sample size. Nonetheless, for a single equation estimation of a Taylor rule for the small open economy I follow Clarida et al. (1998) in choosing GMM. For the remaining parameters, I obtain estimates using simulated method of moments (SMM) estimation. This method is comparable to GMM, but relies on matching moments of the data to those obtained from simulation of the model economy. It is also comparable to the Bayesian approach, as I use prior information before estimation. In this understanding, calibrating a subset of the parameters equals choosing fixed priors, while restricting the range of possible values for the parameters to be estimated equals choosing a uniform prior distribution on a specified domain. As Canova (2007) notes, the approach is easy to implement and, as holds true for small scale models, its functioning is easy to understand. With the procedure described, I check in how far the chosen small scale open economy model can match a set of first and second moments of international data.

1.3 Outline of the Thesis

This dissertation consists of four chapters and an appendix. After this introductory chapter, Chapter 2 is dedicated to the topic of macroeconomic policy in a heterogeneous monetary union. Chapter 3 investigates the macroeconomics of real estate, and Chapter 4 deals with a New Keynesian open economy model versus the six major puzzles in International Macroeconomics. An appendix provides some derivations and

[17] Hansen and Heckman (1996, p. 93).

mathematical details. In the following, I briefly sketch the approach and main findings of Chapters 2 to 4.

Chapter 2 uses a two-country model with a central bank maximizing union-wide welfare and two fiscal authorities minimizing comparable, but slightly different country-wide losses. The rivalry between the three authorities is analyzed in seven static games. Comparing a homogeneous with a heterogeneous monetary union, welfare losses relative to the social optimum are found to be significantly larger in a heterogeneous union. The best-performing scenarios are cooperation between all authorities and monetary leadership. Cooperation between the fiscal authorities is harmful to both the whole union's and the country-specific welfare.

The goal of Chapter 3 is to investigate whether or not it is possible to explain the house price to GDP ratio and the house price to stock price ratio as being generally constant, deviating from its respective mean only because of shocks to productivity? Building a two-sector RBC model for residential and non-residential capital with adjustment costs to capital in both sectors, it is shown that an anticipated future shock to productivity growth in the non-residential sector leads to a large increase in house prices in the present. This property of the model is used to explain the current house price behavior in the U. S., the U. K., Japan and Germany.

In Chapter 4, the following question is posed: Can the New Keynesian Open Economy Model by Galí and Monacelli (2005b) explain "Six Major Puzzles in International Macroeconomics", as documented in Obstfeld and Rogoff (2000*b*)?
The model features a small open economy with complete markets, Calvo sticky prices and monopolistic competition. As extensions, I explore the effects of an estimated Taylor rule and additional trade costs. After translating the six puzzles into moment conditions for the model, I estimate the five most effective parameters using simulated method of moments (SMM) to fit the moment conditions implied by the data. Given the simplicity of the model, its fit is surprisingly good: among other things, the home bias puzzles can easily be replicated, the exchange rate volatility is formidably increased and the exchange rate correlation pattern is relatively close to realistic values. Trade costs are one important ingredient for this finding.

2 Macroeconomic Policy in a Heterogeneous Monetary Union

with Oliver Grimm

We use a two-country model with a central bank maximizing union-wide welfare and two fiscal authorities minimizing comparable, but slightly different country-wide losses. We analyze the rivalry between the three authorities in seven static games. Comparing a homogeneous with a heterogeneous monetary union, we find welfare losses relative to the social optimum to be significantly larger in the heterogeneous union. The best-performing scenarios are cooperation between all authorities and monetary leadership. Cooperation between the fiscal authorities is harmful to both the whole union's and the country-specific welfare.

2.1 Introduction

A country participating in a currency union has to abstain from sovereign monetary policy. A union-wide central bank conducts monetary policy for the whole currency area and cannot pay individual attention to every specific country in its decision-making. In contrast, national fiscal policies typically care about their single country and not the union as a whole. This gives rise to a variety of possible strategic behaviors: National fiscal policies can help monetary policy to maximize union-wide welfare (Benigno 2004; Galí and Monacelli 2002, 2005a), they can try to adjust the outcomes of monetary policy to maximize nationwide welfare (Dixit 2001; Uhlig 2002), or they can be used to maximize the probability of the current government staying in office after the next elections (Beetsma and Uhlig 1999).

Here, a model is proposed that allows to incorporate all three possibilities. We consider a two-country model with a single currency and one monetary policy conducted by a common central bank. Each country or region has its own fiscal policy authority that maximizes its objective function with the arguments of output and inflation. The equations of the basic model and the loss functions are derived from microfoundation by enhancing and modifying the approach of Dixit and Lambertini (2003*a*) and Dixit and Lambertini (2003*b*). Our contribution is to accurately model the possibility of various differences between two countries in a *heterogeneous monetary union*. After setting up the completely symmetric benchmark results, the two regions of the monetary union are allowed to differ from each other with respect to (a) size, (b) home bias, (c) price rigidities and (d) fiscal policies. We take the view that national fiscal policy authorities are concerned with national output and inflation targets, whereas they are not directly concerned with output growth and price changes in other parts of the union unless they decide to cooperate. A crucial assumption for our analysis is that fiscal authorities have target rates for output and inflation that are higher than the welfare-optimal rates. However, monetary policy is assumed to aim at union-wide optimal rates in terms of welfare.

We analyze the fiscal policy makers' and central bank's losses in various scenarios: Policies can be conducted under discretion, simultaneously in the Nash scenario, or sequentially in Stackelberg leadership scenarios for each policy. Alternatively, policies can be coordinated between some or all authorities. We investigate the implications for output, inflation, and various policy loss functions in a numerical analysis, and show that the ranking of the scenarios is relatively robust across different degrees of heterogeneity.

We find that from the viewpoint of welfare maximization, joint cooperation between all policy makers and monetary leadership produce the smallest losses. Increasing the heterogeneities between the regions implies larger overall losses. Finally, we show that the larger the heterogeneities, the higher the relative gains from a first mover advantage of monetary policy.

The literature on monetary and fiscal policy in a monetary union is vast, so we only refer to articles of special importance for our paper.[1] Dixit and Lambertini (2003*b*) consider monetary-fiscal policy interactions in a monetary union. They assume that the participating regions and their policy goals are symmetric and in line with the common central bank's target. Accordingly, optimal output and inflation levels can be achieved – even without coordination of the fiscal authorities and the common central bank

[1] We refer the reader to the textbook by De Grauwe (2003) for an overview of the field, as well as for references to less recent literature.

and without the need for monetary commitment. Dixit (2001),Dixit and Lambertini (2003*b*) and Lambertini (2004, 2006a) check the implications of this model for the case where monetary policy is conservative in the sense of Rogoff (1985). One of their major findings is that fiscal discretion destroys the positive effect of monetary commitment, while fiscal cooperation typically leads to less efficient outcomes than discretionary fiscal policies.

Lombardo and Sutherland (2004) construct a symmetric, two-country model that features government spending in the utility function. They find that the last result can be overturned if the share of steady-state government spending in output is positive and supply shocks are not perfectly negatively correlated. Nonetheless, for plausible parameter values the welfare gains of fiscal cooperation are small.

Dixit and Lambertini (2001) allow for some heterogeneities by assuming that fiscal and monetary authorities may have conflicting output and inflation goals. They show that without commitment or leadership by either authority the ideal points of output and inflation cannot be attained.

Chari and Kehoe (2004) take a closer look at the desirability of fiscal debt constraints. They find that such constraints are undesirable if monetary commitment is possible, whereas the opposite holds if the central bank cannot commit to its policy. The latter is the result of a time-inconsistency problem of monetary policy, which leads to free-riding behavior by the fiscal authorities.

In the very recent literature, the topic of monetary and fiscal interactions has also been dealt with in dynamic, stochastic general-equilibrium models. However, the emphasis in most of these papers is not so much on strategic behavior and game-theoretical scenarios. Gali and Monacelli (2005a) e.g. analyze optimal fiscal and monetary policies in a monetary union where all policy agents care about union-wide variables, and Ferrero (2005) considers a two region model and compares the optimal policies to simple policy rules, where all policy agents care about union-wide variables. Canzoneri et al. (2005) study the interactions between monetary and fiscal policy in a monetary union and compare the results of their New Keynesian model with the data. They also assess the effects of regional asymmetries on welfare, but they assume that fiscal policy is described by exogenously given processes for government spending and distortionary taxes.[2] Lambertini (2006b) attempts to combine the game theoretical approach of the static models with features of dynamic models. To do so, she assumes that fiscal au-

[2]As alternative specifications they consider fiscal policy rules making movements in the budget deficit lead to reactions either in government spending or in tax rates. In our model, by contrast, the government budget is always balanced.

thorities can commit to their policies. Also, she assumes that government spending is exogenously given.

In a series of papers, van Aarle, Engwarda, Plasmans, and Weeren (2001); van Aarle et al. (2002),Engwerda et al. (2005) and Garretsen, Moons, and van Aarle (2005) focus on macroeconomic policy interactions of national fiscal policies and the monetary policy of a common central bank by using a New Keynesian framework. Of these papers, van Aarle et al. (2002) is the one most closely related to our model. They compare the outcomes of different scenarios by distinguishing between non-cooperation, partial cooperation, and full cooperation between monetary and fiscal policies. They find that the stability of coalitions depends strongly on the policy makers' preferences. When the countries are very heterogeneous, non-cooperative behavior is the most likely outcome.

The remainder of the paper is structured as follows. Section 2.2 presents the model, Section 2.3 the various policy scenarios and Section 2.4 parameterization, evaluation method, results, and the sensitivity analysis. The final section concludes.

2.2 Model

We consider a general-equilibrium monetary model with monopolistic distortions and staggered prices. The model is closely related to Dixit and Lambertini (2003*b*) and Benigno (2004). The economy is inhabited by a continuum of individual monopolistic producers. On the one hand, each producer uses his own labor to produce a single differentiated good. On the other hand, each producer, henceforth referred to as "producer-consumer", derives utility from consuming a bundle of goods and from holding real money balances. There exists a continuum of consumption goods over the unit interval which are imperfect substitutes. There are two regions A and B, with the population on the segment $[0, n)$ belonging to region A and the remaining population belonging to region B, with $0 \leq n \leq 1$.[3]

[3]The two-country setting is taken from Benigno (2004). Other related models are Lombardo and Sutherland (2004), Ferrero (2005), and Galí and Monacelli (2005b). In general, our model can be traced back to the seminal work of Blanchard and Kiyotaki (1987) and Obstfeld and Rogoff (1996, Chapter 10).

2.2.1 The Problem of a Producer-Consumer

A producer-consumer j in region $i \in \{A, B\}$ derives utility from aggregate consumption C^j, real money balances M_i^j/P^i and labor N_i^j according to the following function:

$$U_i^j = \left(\frac{C^j}{\gamma}\right)^{\gamma} \left(\frac{M_i^j/P^i}{1-\gamma}\right)^{1-\gamma} - \frac{(N_i^j)^{\beta}}{\beta}, \quad \gamma \in (0,1),\ \beta \geq 1. \tag{2.1}$$

Labor contributes negatively to the utility of agent j, with $\beta - 1$ being the elasticity of marginal disutility of labor. Individual production is assumed to be a function of labor and stochastic productivity,

$$Y_i^j = A_i N_i^j. \tag{2.2}$$

Using the production function to replace labor in the utility function, one obtains

$$U_i^j = \left(\frac{C^j}{\gamma}\right)^{\gamma} \left(\frac{M_i^j/P^i}{1-\gamma}\right)^{1-\gamma} - \left(\frac{\xi_i}{\beta}\right)(Y_i^j)^{\beta}, \tag{2.3}$$

where $\xi_i = A_i^{-\beta}$ captures the fluctuations in region-specific total factor productivity. Changes in this variable may be interpreted as changes in (regional) technology. The total consumption of agent j – who for reasons of exposition is assumed to live in region A – is given by[4]

$$C^j \equiv \frac{(C_A^j)^{\nu^A}(C_B^j)^{1-\nu^A}}{(\nu^A)^{\nu^A}(1-\nu^A)^{1-\nu^A}}, \tag{2.4}$$

where ν^A is a preference shifter of at least the relative size of region a, given by the fixed parameter n: $n \leq \nu^A \leq 1$. Hence, $\nu^A > n$ captures a bias in consumption towards domestic goods .

Consumption of goods from each region is given by

$$C_A^j = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n c^j(a)^{\frac{\theta-1}{\theta}} da\right]^{\frac{\theta}{\theta-1}}, \quad C_B^j = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 c^j(b)^{\frac{\theta-1}{\theta}} db\right]^{\frac{\theta}{\theta-1}}, \tag{2.5}$$

where a is a generic good produced in region A, b a generic good produced in region B, and $\theta > 1$ the elasticity of substitution between different goods in the same region.[5]

[4] For an agent j living in region B, total consumption is given by $C^j \equiv \frac{(C_B^j)^{\nu^B}(C_A^j)^{1-\nu^B}}{(\nu^B)^{\nu^B}(1-\nu^B)^{1-\nu^B}}$ for all $j \in [n, 1]$.

[5] The weights $(1/n)^{(1/\theta)}$ and $(1/(1-n))^{(1/\theta)}$ are a "normalization with the implication that an increase in the number of products does not affect marginal utility after optimization". See Blanchard and Kiyotaki (1987, p. 649)

The elasticity of substitution of the home and foreign bundles of goods equals one. The corresponding consumer price indices – with subscripts denoting the place of production and superscripts denoting variables specific to agent j or region i – are

$$P^A \equiv (P_A^A)^\nu (P_B^A)^{1-\nu} \qquad \text{and} \qquad P^B \equiv (P_B^B)^\nu (P_A^B)^{1-\nu}, \tag{2.6}$$

where the respective elements are given by

$$P_A^i \equiv \left[\frac{1}{n}\int_0^n p^i(a)^{1-\theta} da\right]^{\frac{1}{1-\theta}} \quad \text{and} \quad P_B^i \equiv \left[\frac{1}{1-n}\int_n^1 p^i(b)^{1-\theta} db\right]^{\frac{1}{1-\theta}} \tag{2.7}$$

and denote the market-price indices of goods consumed in region i and produced in region A and B, respectively. The small letter prices $p(a)$ and $p(b)$ denote the price set by a generic producer-consumer in region A and B, respectively. These prices are chosen as to maximize the indirect utility function. When setting her price, the producer-consumer is faced a certain type of price rigidity, such that only a fraction of prices can be adjusted after shocks hit the economy. Details of price setting are presented in appendix Section 5.1.2.

The price index P^i is defined as the minimum expenditure necessary for purchasing goods leading to a consumption index C^j of size one, and the price indexes P_A^i and P_B^i are defined as the minimum expenditure required to purchase goods resulting in consumption indexes C_A^j and C_B^j, which equal one.

We assume goods-market arbitrage leads to identical prices across borders such that $P_A^A = P_A^B = P_A$ and $P_B^A = P_B^B = P_B$. This implies that the i superscripts on each left hand side of Equation (2.7) can be dropped, and the incentive to set different prices across regions because of the home bias in consumption bears no consequences.[6] So the price level of goods consumed in region $i \in \{A, B\}$ – region i's consumer price level – given in Equation (2.6) simplifies to

$$P^i = (P_i)^\nu (P_{-i})^{1-\nu}, \tag{2.8}$$

where $-i$ denotes the opposite region than i. Again, a superscript refers to the place of consumption and a subscript to the place of production. So Equation (2.8) states that region i's CPI is a combination of the indexes of goods produced in region i and in region $-i$. Denoting the output of producer-consumer j in region i by Y_i^j, the budget

[6]In our model, inflation differentials occur due to the home-bias effect, as the composition of the consumption bundles differ in both regions. This assumption is somewhat critical when referring to the Euro-zone, where significant price differences for the same product in different countries exist even for tradeable goods.

constraint for this agent is

$$\int_0^n p^i(a)c^j(a)da + \int_n^1 p^i(b)c^j(b)db + M_i^j = p^i(j)Y_i^j(1-\tau_i) - P_i T_i + \bar{M}_i^j \equiv I_i^j. \quad (2.9)$$

The budget constraint guarantees that the sum of consumption expenditures plus money demand equals nominal net income I_i^j, which is the sum of sale revenues from the good produced and beginning-of-period money holdings minus net tax payments.

Macroeconomic policy consists of three elements. A common central bank chooses the nominal money supply and in each of the two regions, a fiscal authority uses its tax rate τ_i proportional to sales to subsidize production.[7] The government is not allowed to be indebted; its budget is balanced by lump-sum taxes T_i. For the two regional government budget constraints we have

$$\begin{aligned} \int_0^n p^A(j)y(j)\tau_A dj + nP_A T_A = 0 \\ \int_n^1 p^B(j)y(j)\tau_B dj + (1-n)P_B T_B = 0 \end{aligned} \quad (2.10)$$

2.2.2 Terms of Trade and Equilibrium

As set out before, the law of one price holds in the economy considered, i.e. $p^A(a) = p^B(a)$ and $p^A(b) = p^B(b)$. Nonetheless, agents appreciate consumption of domestically produced goods more. Hence, the (consumer) price index in region A, P^A includes a larger share of domestic goods than the (consumer) price index in region B, P^B. This has implications for the terms of trade, which we define as follows.

Definition 1. *The terms of trade for region i, S_i, are given by the price of imports relative to the price of exports. Using "$-i$" to denote "not i",*

$$S_i \equiv \frac{P_{-i}^i}{P_i^{-i}}. \quad (2.11)$$

Here, P_{-i}^i is the price level of goods produced in region $-i$ and consumed in region i, i.e. imports, whereas P_i^{-i} is the price level of goods produced in region i and consumed in region $-i$, i.e. exports. E. g., region A's terms of trade are given by the price levels

[7] This assumption for fiscal policy is typically used in New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models.

of goods produced in region B and consumed in region A, divided by the price level of goods produced in region A and consumed in region B.[8] The following lemma applies.

Lemma 1. *The terms of trade are equal to the ratio of producer price indices.*

$$S_i = \frac{P_{-i}}{P_i}. \tag{2.12}$$

Proof. *The equality holds as the rate of substitution between goods of region i is constant in both economies, so that the basket of domestically produced goods has the same composition in both economies, though not the same relative size. Therefore, a change in the price index of goods produced in region i has the same impact on e.g. P_i^{-i}, the price index of region i produced goods consumed in region $-i$, and on P_i^i, and we can drop the superscript.* ■

Using the definitions of the consumer price indices given in equation (2.8), we can relate the terms of trade to the consumer price indices P^A and P^B and to the price indices of goods produced in each region, P_A and P_B as follows:

$$\frac{P^A}{P_A} = (S_A)^{1-\nu}, \quad \frac{P^A}{P_B} = \frac{1}{(S_A)^{\nu}}, \quad \frac{P^B}{P_A} = (S_A)^{\nu} \quad \text{and} \quad \frac{P^B}{P_B} = \frac{1}{(S_A)^{1-\nu}}. \tag{2.13}$$

In the case of an identical home bias in both regions, which we are assuming here, the ratios of the two measures of inflation are inversely related to each other:[9] $S_i = 1/S_{-i}$. Movements in the terms of trade imply movements in relative prices and, therefore, shift demand across the border.

Definition 2. *Given policy decisions for M and τ_i, an equilibrium is an allocation $\{C, (C_i)_{i\in\{A,B\}}, (C_i^j)_{j\in[0,1]}, Y, (Y_i)_{i\in\{A,B\}}, (Y_i^j)_{j\in[0,1]}\}$ and a price system $\{P, (P_i)_{i\in\{A,B\}}, (P^i)_{i\in\{A,B\}}, (p_i^j)_{j\in[0,1]}\}$, such that*

1. *the allocation maximizes the utility of the producer-consumer,*
2. *markets clear,*
3. *the policies are consistent with allocation and prices.*

The equilibrium of the model is derived in the appendix, in Sections 5.1.1 to 5.1.3. Together with the decisions of monetary and fiscal policies, it can be represented in two

[8] It follows that the terms of trade for region B, $S_B = P_A^B / P_B^A$ are the inverse of S_A. Note that the usual definition, see e. g. Obstfeld and Rogoff (1996, p. 242), is in line with ours from the viewpoint of region B.

[9] See Galí and Monacelli (2002) for a similar treatment in a small open economy setting.

equations, an equation relating output to real money holdings,[10]

$$Y_A = \gamma \frac{\bar{M}}{P} \frac{\gamma}{1-\gamma[\nu + (1-\nu)\frac{1-n}{n} S_A]} \qquad \text{and} \qquad Y_B = \gamma \frac{\bar{M}}{P} \frac{1}{1-\gamma[\nu + (1-\nu)\frac{n}{1-n} S_B]}, \tag{2.14}$$

and a price rule for an individual producer-consumer,[11]

$$\left(\frac{p(a)}{P_A}\right) = \left(\frac{\theta \xi_A}{(\theta-1)(1-\tau_A)} Y_A^{\beta-1}\right)^{\frac{1}{1+\theta(\beta-1)}} \quad \text{and} \quad \left(\frac{p(b)}{P_B}\right) = \left(\frac{\theta \xi_B}{(\theta-1)(1-\tau_B)} Y_B^{\beta-1}\right)^{\frac{1}{1+\theta(\beta-1)}}. \tag{2.15}$$

The variable $\bar{M}/P$ denotes aggregate beginning-of-period real money holdings which are assumed to be identical across agents and regions. For detailed derivations of both equations the reader is referred to the appendix.

2.2.3 Analysis

So far, the model's time dimension is fairly simple. At the beginning of the period, shocks did not yet occur and the economy is at its steady state. Then, the shocks occur and all adjustments take place. As we are interested in exactly these deviations from steady state, we approximate our solution of the model around a steady state with identical price levels. We denote the percentage deviation of a price level from its steady state as inflation rate and use the small letters y and s to denote the percentage deviation of output and terms of trade from its respective steady state. A log-linear approximation to the model equilibrium is given by the following two propositions.

Proposition 1. *In equilibrium, inflation of region i is related to the change in money supply and the deviations of the domestic and the foreign tax rate from their respective steady states. It is also related to private expectations about the model variables and to actual and expected stochastic technology, all subsumed in the variable ψ_i:*

$$\pi_i = d^i \hat{m} + c^i \hat{\tau}_i + c^{-i} \hat{\tau}_{-i} + \psi_i, \quad i \in \{A, B\}. \tag{2.16}$$

Proof. *See Section 5.1.5 in the appendix.*

The formulation used here shows how all three policy authorities affect a regional inflation rate. The central bank influences the policy variable $\hat{m}$, the change in the beginning-of-period real money holdings. As one would expect, an expansionary mon-

[10] This is Equation (5.29) in the appendix.
[11] This is Equation (5.18) in the appendix, using Equation (5.26) to plug in regional output.

etary policy, i.e., an increase in the real money supply ceteris paribus increases inflation. In our calibration, the parameter d^i is of positive sign.

The parameter c^i refers to the influence of national fiscal policies on inflation, and c^{-i} measures the spill-over effect from foreign fiscal policy on region i's inflation. Both parameters are complex combinations of the model's structural parameters and steady state values, so we abstain from stating them here and refer the reader to Section 5.1.5 in the appendix. Both c^i and c^{-i} have typically negative signs: Dixit and Lambertini (2003*a*) indicate that the sign of the parameters may become negative when tax cuts and subsidies raise the supply of goods. The absolute value of c^i is higher than that of c^{-i}, i. e., direct effects from fiscal policies are stronger than the resulting spill-over effects to the other region. The implied values for our benchmark structural parameters are presented in Table 2.1.

Proposition 2. *The deviation of region i's output from its steady state is related to changes in the domestic as well as in the foreign tax rate, to domestic surprise inflation, to the terms of trade and to changes in the productivity differential between the domestic and the foreign region. It is given by*

$$y_i = a^i \hat{\tau}_i + a^{i,-i} \hat{\tau}_{-i} + b^i (\pi_i - \pi_i^e) + \kappa^i s_i + \phi_i, \tag{2.17}$$

where $a^i \equiv \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)} \right) \bar{\tau}_i$ *captures the effect of the home country's fiscal policy instrument and* $a^{i,-i} \equiv -\left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)} \right) \bar{\tau}_{-i}$ *the effect of foreign fiscal policy on domestic output.*

Proof. *See Section 5.1.6 in the appendix.*

Assuming that fiscal policies choose the steady-state level of taxes $\bar{\tau}_i$ optimal in order to offset the monopolistic distortion, $\bar{\tau}_i$ will be negative. Therefore, an expansionary fiscal policy is given if $\tau_i < \bar{\tau}_i$, i.e., if $\hat{\tau}_i = \frac{\tau_i - \bar{\tau}_i}{\bar{\tau}_i} > 0$. It is important to keep this in mind to follow the fiscal policy description in Section 2.3. Additionally, fiscal policies have positive spill-over effects onto the other region. Therefore, both a^i and $a^{i,-i}$ have a positive sign.

The effect of domestic surprise inflation on output is captured by $b^i \equiv \frac{2\beta\rho}{(\beta-1)(1-\rho)}$, where the parameter $\rho \in \{0,1\}$ determines the degree of price stickiness, from flexible prices ($\rho = 0$) of all goods to fixed prices of all goods for $\rho = 1$. In line with the landmark papers by Kydland and Prescott (1977) and Barro and Gordon (1983), surprise inflation generates an increase in the national output level, as b_i has a positive sign. The private sector has rational expectations about inflation, i. e. the following condition holds:

$$\pi_i^e = E(\pi_i). \tag{2.18}$$

The terms of trade effect on regional output is captured by $\kappa^i \equiv \frac{\beta\rho}{(\beta-1)(1-\rho)} > 0$. Region i's terms of trade s_i are given by the log-linear approximation of Equation (2.12):

$$s_i = (\pi_{-i} - \pi_i)\ . \tag{2.19}$$

We know from empirical studies that the terms of trade effect also depends on the region's size. This means that a smaller region typically has a higher κ^i, implying that inflation differentials have a greater effect on output, something that is missing here.[12] A higher inflation rate in region $-i$ than in region i corresponds to a real depreciation of region i and thus increases its net exports. This shift of consumption from foreign goods (region $-i$) to domestic goods (region i) increases domestic income. As for the inflation equation, Table 2.1 shows the implied values for our benchmark structural parameters.

Finally, a random shock ϕ_i enters the output equation, which is an i.i.d. shock with zero mean and a variance $\sigma^2_{\phi_i}$. In the appendix we show that this shock is the weighted difference of the deviations of the two regional (stationary) productivity processes from their respective steady states, as given by

$$\phi_i = \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\hat{\xi}_i - \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)\hat{\xi}_{-i}.$$

Henceforth, ϕ_i is denoted as the "region-specific" output shock. In the following section we will focus attention on the equations given in Proposition 1 and 2, which summarize the microeconomic model.

2.3 Policy Analysis

This section has two main goals. First, we state our assumptions on the maximization problems of fiscal and monetary authorities. These assumptions are based on a

[12] Additionally, we assume that the intensity of trade inside the currency area is high enough for effects from outside the union to be neglected. Another possibility for eliminating outside effects is to assume that all regions within the monetary union have similar trade relations with the rest of the world, such that these are negligible for our results.

"quadratic loss function that represents a quadratic (second-order Taylor series) approximation to the level of expected utility of the representative household in the rational expectations equilibrium associated with a given policy.´´[13] This method, developed and well described in Woodford (2001) and Woodford (2003, Chapter 6) is widely used in the literature. In accordance with these sources, we assume that nonlinear effects on welfare are negligible. If this were not the case, the validity of the method would be restricted, as Woodford (2003, p. 386ff.) points out. Then, one could rely on higher-order algorithms, as exemplified in Kim, Kim, Schaumburg, and Sims (2005). In Sections 5.1.7 and 5.1.8 in the appendix we fully derive a second-order approximation to welfare of one region and of the monetary union as a whole.

2.3.1 Suboptimal Target Functions of Fiscal Authorities

Fiscal authorities minimize a quadratic loss function that aims at national inflation and national output. The functional form of the loss function is identical to that of regional welfare, as derived in Section 5.1.7 in the appendix.

$$L_{F^i} = \frac{1}{2}\left[(\pi_i - \pi_F^i)^2 + \theta_F^i (y_i - y_F^i)^2\right] . \tag{2.20}$$

Here, π_F^i is the fiscal policy's inflation target in region i, and y_F^i is the desired output level of the fiscal authority in region i. According to the utility-based welfare criterion, these reference values should be equal to zero for inflation and to the flexible price output plus the steady state deviation from the efficient steady state in the case of output.[14] If both fiscal authorities and the monetary authority agree on these welfare-maximizing targets, the first-best situation with the highest possible welfare can be obtained, "despite disagreements about the weights of the objectives, despite ex post monetary accommodation to fiscal profligacy, without fiscal coordination, without monetary commitment, and for any order of moves": this is demonstrated in Dixit and Lambertini (2003*b*). It corresponds to the joint cooperation case in our model, discussed later.

However, EMU national governments and the ECB have often disagreed about the appropriate strategy for their policies. And once there is a disagreement about optimal target levels, policy interactions become more relevant – and much more interesting. Therefore, we choose to model a discrepancy in the target levels between fiscal and monetary authorities. In particular, we make the following assumption:

[13] Woodford (2003, p. 383).

[14] With some simplifying assumptions, the optimal target for output is also zero.

Each fiscal policy authority does not maximize regional welfare, but instead chooses target values for output and inflation that are slightly above the socially optimal levels. There may be various reasons for this assumption. It may be justified by the fiscal policy makers' desire to attain greater government size (cf. Fatás and Rose 2001), or by their incentive to maximize reelection probability (cf. Beetsma and Uhlig 1999). To illustrate the latter case, one can imagine that fiscal authorities are able to deceive their voters about the socially optimal targets, particularly during election campaigns. This would be especially true of a monetary union, where fiscal policy communicates with the *domestic* society, while monetary policy is *centralized* and concerned with the whole society of the monetary union. Accordingly, it communicates with the private sector of each individual region from a greater distance. Finally, another possible explanation for differences in the target values is a simple one-time mismeasurement.

Furthermore, we assume that the target values of fiscal policy authority in region A may differ from the ones of the fiscal policy authority in region B. Economically intuitive reasons for considering different inflation targets on the part of the agents may be given (i) by home-bias effects in the consumption of goods, (ii) by different elasticities of substitution in the representative agents' utility function across regions, or (iii) by different proportions of tradeable and non-tradeable goods in both regions. In our microeconomic model we have incorporated a home-bias effect in consumption and considered region-specific productivity shocks, which represent possible reasons for different fiscal targets in the two regions.

To obtain a microfoundation of the fiscal target function (2.20) with targets that are suboptimal from a social welfare perspective, one could assume as mentioned before that the government derives additional utility of government size. To be more precise, government utility could be given as a weighted sum of the representative agent's utility and additional utility from a greater government size. The fiscal authority receives this extra utility if Y^i exceeds the social optimal level:

$$L_F^i = \frac{Q}{2}\left[(\pi_i - \bar{\pi}_i)^2 + \theta^i (y_i - \bar{\bar{y}}_F^i)^2\right] + \frac{1-Q}{2}(y_i - \bar{\bar{y}}_F^i + \iota)^2 \,, \tag{2.21}$$

where $\iota > 0$ and $\bar{\bar{y}}_F^i$ denotes the socially optimal output and $\bar{\pi}_i$ socially optimal inflation, as derived in Equation (5.102) in Section 5.1.7 in the appendix. As soon as the weight on this additional utility $0 < Q < 1$ is nonzero and positive, the fiscal authority will have different target values in its loss function. Then, this loss function can be rewritten as

$$L_F^i = \frac{1}{2}\left[(\pi_i - \pi_F^i)^2 + \theta_F^i (y_i - y_F^i)^2\right], \tag{2.22}$$

where $y_F^i > \bar{\bar{y}}_F^i$ and $\theta_F^i > \theta_i$.

2.3.2 Optimal Target Function of the Common Central Bank

We assume the common central bank *maximizes the union-wide social welfare function*, as derived in Sections 5.1.7 and 5.1.8 in the appendix. Using a notation with the index M to denote monetary policy, we have

$$\begin{aligned} L_M =& \frac{1}{2}\left[n\left((\pi_A - \pi_M^A)^2 + \theta_M^A (y_A - y_M^A)^2\right)\right. \\ &+ \left.(1-n)\left((\pi_B - \pi_M^B)^2 + \theta_M^B (y_B - y_M^B)^2\right)\right] . \end{aligned} \tag{2.23}$$

In the case of excessive fiscal targets, as motivated above, we can state that the central bank is relatively conservative in comparison to fiscal policies, given by $\pi_M^i < \pi_F^i$ and $y_M^i < y_F^i$ for all i. Our model differs in that respect from the approach of Dixit and Lambertini (2003*b*): They assume that fiscal policies act in a socially optimal manner and the central bank is too conservative, whereas we claim that the central bank maximizes union-wide welfare and fiscal policies act in too expansionary a way.

The different weights on output stabilization and the different output and inflation targets of monetary and fiscal policies give rise to trade-offs among policy makers. Whereas the fiscal authorities attach greater importance to output stabilization (and to pushing output and inflation above their natural levels), the common central bank sets a relatively higher weight on stabilization of inflation. These conflicting targets induce strategic behavior among the policy makers, which is examined in the following.

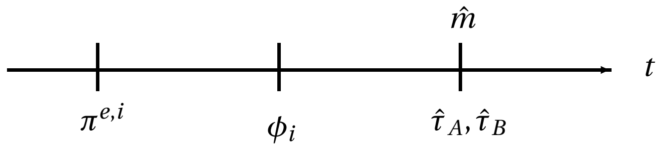
2.3.3 Scenarios of Simultaneous Decision-Making

In this subsection, we consider the scenario in which both fiscal authorities and the common central bank choose their optimal policies simultaneously. We evaluate the outcomes of the scenarios numerically in Section 2.4.

Nash Behavior

First, we consider the scenario of not cooperating fiscal and monetary policies. The policy makers decide upon their optimal policies after having observed the realizations of the region-specific shocks. Thus, they take the households' expectations on inflation as given. For better understanding, the sequence is depicted in Figure 2.1.

Figure 2.1: Time Structure for Simultaneous Decision-Making (with $i = A, B$)



Country A's fiscal policy maker optimizes the loss function (2.20) with respect to $\hat{\tau}_A$, while taking the decision of the other region's fiscal policy, $\hat{\tau}_B$, and the policy choice of the common central bank, $\hat{m}$, as given. Accordingly, country B optimizes (2.20) with respect to $\hat{\tau}_B$, while taking the policy choices of fiscal policy in country A ($\hat{\tau}_A$) and that of the common central bank ($\hat{m}$) as given.

Simultaneously, monetary policy optimizes the union-wide social loss function (2.20), taking the fiscal policy actions and the expectations of the private sector as given.

Cooperation of Monetary and Fiscal Policies

According to many economists and politicians, cooperation between policies plays a crucial role for heterogeneous agents. This is emphasized by the fact that regions and international organizations create institutions like the Stability and Growth Pact and aim at further common targets like tax harmonizations, which are only a few examples of a cooperative policy device. In this subsection, we analyze the scenario of cooperation under discretion characterized by an agreement of the political authorities on common policy goals, i. e. $\pi_F^A = \pi_F^B = \pi_M = \pi_{JC}$, $y_F^A = y_F^B = y_M = y_{JC}$ and $\theta_F^A = \theta_F^B = \theta_M = \theta_{JC}$, where the subscript JC denotes the "joint cooperation" scenario. The timing of political decision-making corresponds to the Nash scenario and is illustrated in Figure 2.1. We assume here, that the policy makers share a combined loss function of the following kind:

$$
\begin{aligned}
L_{JC} = & n\frac{1}{2}[(\pi_A - \pi_{JC})^2 + \theta_{JC}(y_A - y_{JC})^2] \\
& + (1-n)\frac{1}{2}[(\pi_B - \pi_{JC})^2 + \theta_{JC}(y_B - y_{JC})^2] \ .
\end{aligned} \tag{2.24}
$$

The minimizing problem follows the same pattern as in the Nash scenario, the only difference being that all authorities face the same loss function. We implicitly treat the joint cooperation case as if the policy makers were committed to the socially optimal targets, i.e. *we assume that all policy makers aim at attaining the social optimum in this scenario and that the private sector is aware of that when forming its expectations about inflation.* We do not incorporate possible deviations from this strategy, though this could be an interesting enhancement of this model. Thus, the first-best optimum for the private agents is attainable under joint cooperation. Dixit and Lambertini (2003*b*) use the same assumption in their model. We return to this point in Section 2.4.

Independent Monetary Policy and Cooperation between Fiscal Policies

If fiscal policy makers decide to cooperate while monetary policy acts independently, the fiscal authorities optimize a similar loss function as in the joint cooperation scenario. The loss function differs in the target values of inflation and output above the socially optimal levels. The fiscal objective function of both regions is given by

$$
\begin{aligned}
L_{FC} = & n\frac{1}{2}\left[(\pi_A - \pi^A_{FC})^2 + \theta_{FC}(y_A - y^A_{FC})^2\right] \\
& + (1-n)\frac{1}{2}\left[(\pi_B - \pi^B_{FC})^2 + \theta_{FC}(y_B - y^B_{FC})^2\right] \ ,
\end{aligned} \tag{2.25}
$$

where the subscript FC denotes "fiscal cooperation". The monetary authority optimizes the loss function (2.20). The solution is obtained analogously to the previous cases.

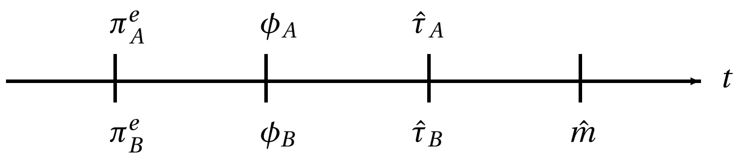
2.3.4 Scenarios of Sequential Decision-Making

The policy choices made by monetary and fiscal authorities may possibly take place at different times due to certain pre-scheduled rules, bureaucracy, or special intrinsic features of the political institutions. Therefore, we focus here on interactions between fiscal and monetary policies when both authorities act sequentially. The evaluation of the different scenarios follows in Section 2.4.

Stackelberg Leadership of Fiscal Policy

We begin with the scenario of fiscal leadership, i. e. fiscal policy makers have to decide on their policy actions before monetary policy has been implemented and after having observed the realization of the regional shocks ϕ_i. Thereby, they take the household's inflation expectation as given. Beetsma and Bovenberg (1998) argue that fiscal leadership seems to be more likely when monetary policy can be implemented and adjusted more quickly than fiscal policy. This may be applicable when choices for taxes and subsidies are accompanied by bureaucratic and legislative processes that provide the fiscal authority with leadership over monetary policy. The sequence in that scenario is depicted in Figure 2.2.

Figure 2.2: Time Structure for Sequential Decision-Making (Fisc. Leadership)

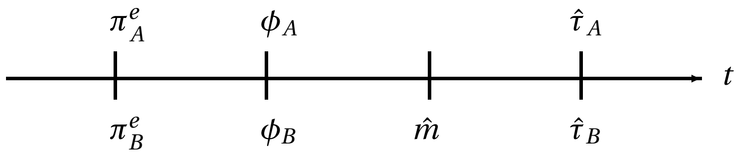


The solution of the game is obtained by backward induction. Solving the monetary policy's optimization problem at the second stage of the game leads to the optimal choice of $\hat{m}$ while taking the fiscal policy variables $\hat{\tau}_A$ and $\hat{\tau}_B$ as given. In the first stage, the fiscal policy maker of region i optimizes $\hat{\tau}_i$ to react to the action taken by the policy maker of region j, $\hat{\tau}_j$, and subject to the monetary reaction function, which is derived from the second stage of the game.

Stackelberg Leadership of Monetary Policy

In contrast to the previous case, monetary policy attains Stackelberg leadership over fiscal policies if it only affects the economy with a lag of time exceeding the legislative and bureaucratic time needed for fiscal policy decision-making. Another reason for monetary leadership could be that monetary policy follows a certain rule. E.g., the ECB pursues, but is not committed to achieve an inflation rate of close to, but below two percent. The timing is shown in Figure 2.3. The solution is similar to the former scenario of fiscal leadership. In the second stage, fiscal policy makers minimize the loss function (2.20) analogously to the Nash scenario shown above, given the other region's fiscal policy and the monetary policy variable $\hat{m}$. The common central bank chooses $\hat{m}$ in the first stage, given the best responses of the fiscal policies $\hat{\tau}_A$ and $\hat{\tau}_B$.

Figure 2.3: Time Structure for Sequential Decision-Making (Mon. Leadership)



Fiscal Cooperation and Sequential Policy Actions

Analogously to the fiscal corporation scenario where the policy makers choose their optimal policies simultaneously, one can also assume cooperation between national fiscal policies when the decision-making on monetary and fiscal policies takes place at different stages. The motivation for cooperating fiscal policies in a sequential policy game corresponds to that of fiscal cooperation in a simultaneous game. Accordingly, we also analyze scenarios (i) *fiscal cooperation when fiscal policy moves first* and (ii) *fiscal cooperation when monetary policy moves first.*

The time structure of scenario (i) corresponds to the one in Figure 2.2, while the time structure of scenario (ii) corresponds to that in Figure 2.3. The optimization problem under both scenarios follows the same pattern as in the corresponding sequential scenarios without cooperation and are, therefore, omitted in this section.

2.4 Results

In the following we derive numerical results for the seven scenarios of strategic behavior between monetary and fiscal authorities introduced in the previous section.

We, first, describe the calibration of the model. Second, we show the evaluation methods used for the ranking of the different scenarios. Third, we run simulations for the case of a homogeneous and a heterogeneous monetary union by using the structural parameters from the microfounded model of Section 2.2. In this case, fiscal policy aims at granting production subsidies and levying per-capita taxes to reduce the distortions caused by monopoly power. We use the results from the homogeneous monetary union as a reference case and compare the rankings of different scenarios in the heterogeneous case. Fourth, we strengthen our results by using a sensitivity analysis of both the structural parameters and the policy targets.

2.4.1 Calibration

We calibrate the structural parameters of the model in accordance with the standard literature, as referred to in Dixit and Lambertini (2003*a*, Appendix F). The elasticity of marginal disutility of labor is set at 0.45, a value proposed by Blanchard and Fischer (1989).[15] This implies that the disutility parameter β, which is one plus the inverse of the elasticity of marginal disutility of labor, has the value $\beta = 3.22$. The Calvo-stickiness parameters Φ^H and Φ^F are set at a moderate value of 0.5, implying an average price to be fixed for three periods. The elasticity of substitution between goods of the same region is set at $\theta = 11$, as in Dixit and Lambertini (2003*a*). Obstfeld and Rogoff (2001) discuss the literature that has found values between 1 and 20. Note that the elasticity of substitution between goods of different regions is set to unity, as in Benigno (2004). In setting the steady state of the technology parameter as $\bar{\xi}_i = 1$ and the subjective discount factor as $\eta = 0.98$ we strictly follow Dixit and Lambertini (2003*a*). The steady-state value for the fiscal policy instrument is assumed to be set optimally, i. e. to offset monopolistic distortion. Via $\bar{\tau}_i = 1/(1-\theta)$ we obtain a subsidy rate of ten percent for both regions in the steady state.

We look here at two different cases. In the first case, both regions have the same size ($n = 1 - n = 0.5$) and are completely symmetric, with identical structural parameters, identical fiscal policies, and no home bias ($v^H = v^F = 0.5$). In the second case, region B accounts for only 30 percent of the union and displays more price rigidities. The latter assumption is based on the findings of Benigno and Lopez-Salido (2004). They estimate the price rigidity in five core EMU countries and identify substantial heterogeneities.[16]

In the second case we presume that there is also a considerable home bias in consumption in both regions, thus following Anderson and van Wincoop (2003).

Given the values stated above, we can calculate the various parameters a^i, b^i, c^i, d^i and κ^i in the model equations. Also, we can infer the values in the policy loss functions maximizing social welfare: In the symmetric case, these are target values for inflation and output, both equal zero, and a weight on output of $\theta_M^A = \theta_M^B = 0.00763$. In the

[15] The authors discuss this parameter on page 341. Dixit and Lambertini (2003*a*) assume unit wage elasticity and thus less curvature.

[16] The average price duration varies between around four quarters in the Netherlands and Germany and up to 17 quarters in Spain, implying price rigidity parameters between 0.75 and 0.94. We will choose numbers between 0.5 and 0.58, following the more conservative estimates of Bils and Klenow (2004). For a closer look at European data, the reader is referred to Dhyne, Alvarez, Le Bihan, Veronese, Dias, Hoffmann, Jonker, Lünnemann, Rumler, and Vilmunen (2005).

asymmetric case, the output weight for region B rises to $\theta_M^B = 0.01046$, while all other socially optimal target values remain the same.

As stated earlier, we assume that the common central bank sticks to these values, while the fiscal policy authorities may deviate from them. There may be various reasons for such deviation, for example systematic mismeasurement by the fiscal authorities or the fiscal authorities maximizing a different objective function they are able to conceal from the households. This was substantiated in Section 2.3. More particularly, we assume that the fiscal policy authorities put equal weight on output and inflation of unity. Furthermore, fiscal policies have higher target values for output $y_F^A = y_F^B = 0.015$ and inflation $\pi_F^A = \pi_F^B = 0.02$. In the asymmetric case, fiscal policy in region B even puts a weight of $\theta_F^B = 1.25$ on output, sets its output target at $y_F^B = 0.025$ and its inflation target at $\pi_F^B = 0.03$, which could be seen as the result of its self-perception as a high-growth, catch-up region. Table 2.1 summarizes this calibration. As in Dixit and Lambertini (2003*a*), the stochastic term is calibrated to match the variance of output around its steady state as plus/minus six percent, as is the case for the U. S.

As set out in Section 2.3, we assume that the private sector has rational expectations about inflation. In our analytical calculations, we treat π_A^e and π_B^e as given. The inflation expectations of the private agents in both countries are determined in our model by iteration. In other words, we use an arbitrary starting value for the inflation expectations in both countries and repeat the optimization calculations until the inflation expectations differ from realized inflation by a value of less than 10^{-10} for both countries, while keeping the shock at its expected value of zero. This approach guarantees that $\pi_i^e = E(\pi_i)$ holds for $i = A, B$. After inflation expectations are determined, we simulate our model by averaging over 100,000 random draws of the stochastic processes.

2.4.2 Evaluation Method

The main purpose of our numerical approach is to rank the different scenarios of strategic behavior of monetary and fiscal policies for the losses they induce. We distinguish three approaches:

(i) Evaluation of the loss functions referring to the policy exercised by the fiscal and monetary authorities. In each cooperation scenario, the corresponding loss function is a compromise between the cooperating authorities.

(ii) Evaluation of the region-specific loss functions. In each cooperation scenario,

Table 2.1: Calibration of the Baseline Model

Parameter	Value*	Alternative*	Explanation
Structural parameters			
n	0.50	0.70	Size of region A
ν	0.50	0.80	Parameter capturing preference for home goods
β	3.22	3.22	One plus one over the elasticity of marginal disutility of labor
Φ	0.50	0.58	Fraction of firms that cannot adjust prices
θ	11.00	11.00	Elasticity of substitution between goods
ξ_i	1.00	1.00	Technology parameter
η	0.98	0.98	Subjective discount factor
$\bar{\tau}_i$	-0.10	-0.10	Steady state value of taxes
Implied macro parameters			
a^i	0.042	0.042	Fiscal policy effect on output
$a^{i,-i}$	0.003	0.003	Foreign fiscal policy effect on output
b^i	8.473	13.098	Monetary policy effect on output
κ^i	4.236	6.549	Terms of trade effect on output
c^i	-0.001	0.005	Fiscal policy effect on inflation
c^{-i}	-0.000	-0.006	Foreign fiscal policy effect on inflation
d^i	0.060	5.474	Monetary policy effect on inflation
Loss functions			
θ_M^i	0.00736	0.01046	Central bank's weighting factor for output gap
π_M^i	0.00	0.00	Inflation target of the central bank
y_M^i	0.00	0.00	Output gap target of the central bank
θ_F^i	1.00	1.25	Fiscal policy's weighting factor for output gap
π_F^i	0.02	0.03	Inflation target of fiscal policy
y_F^i	0.015	0.025	Output gap target of fiscal policy

Remarks: The term "Value" denotes the value chosen for both regions in the symmetric case and for region A in the asymmetric case. "Alternative" denotes the value chosen for region B in the asymmetric case. $i = A, B$.

these are the region-specific loss functions the policy authorities would minimize if they were not cooperating. This approach allows us to infer whether cooperation scenarios are preferable for each participating policy authority individually.

(iii) Evaluation of social welfare. For each region, we calculate the welfare loss that arises due to deviations in output and inflation from the socially optimal values.

We show the losses involved in all three approaches in Table 2.2. In our discussion we incorporate only the second and third approach. The reasoning behind this is as follows: In approach (i), the losses of the three policy authorities are based on the loss functions used in the optimization calculations. If the policy makers decide to cooperate, they usually compromise on targets that differ from their own *true preferences*. However, the "true losses" which the policy makers face are still based on their specific preferences. Therefore, in approach (ii) we calculate the values of the policy makers' loss functions given by equations (2.20) and (2.23), irrespective of the loss function used for optimization in the relevant scenarios.[17] One should also take these losses into account, when exploring whether joint cooperation among all policy makers or cooperation between fiscal policy makers can take place on a voluntary basis.

The region-specific social welfare losses of approach (iii) are given by[18]

$$L_A = \frac{1}{2}((\pi_A - \pi_M^A)^2 + \theta_M^A(y_A - y_M^A)^2)$$
$$L_B = \frac{1}{2}((\pi_B - \pi_M^B)^2 + \theta_M^B(y_B - y_M^B)^2).$$

Additionally, we express the region-specific social losses in terms of an equivalent reduction in region-specific consumption units, following the example of Lucas (2003). A scenario "performs best" when it shows the lowest reduction of consumption units compared to the consumption level in the social optimum. The calculation of the consumption-equivalent losses follows the approach of Adam and Billi (2006).

From our welfare derivation we know that for region A

$$U^A = -\bar{Y}_A u_C L_A \tag{2.26}$$

[17]Note that by this definition the losses in case (ii) only differ from the losses in case (i) for the joint cooperation scenario and the scenarios of fiscal cooperation.

[18]Recall from Section 2.3 that the central bank is assumed to optimize the union-wide social loss, which is a region-sized weighted sum of the social losses of region A and B.

holds. To derive a relation between a permanent reduction of consumption (given by δ_C^A percent) and the welfare loss, a second-order Taylor approximation of the utility loss is generated by

$$U^A \approx \left(-\frac{u_C \bar{Y}_A \delta_C^A}{100} + u_{CC}\left(\frac{\bar{Y}_A \delta_C^A}{100}\right)^2\right) = -u_C \bar{Y}_A \left(\frac{\delta_C^A}{100} - \frac{u_{CC}\bar{Y}_A}{u_C}\left(\frac{\delta_C^A}{100}\right)^2\right)$$
$$= -u_C \bar{Y}_A \left(\frac{\delta_C^A}{100} + \frac{(1-\gamma)\bar{Y}_A}{\bar{Y}_A}\left(\frac{\delta_C^A}{100}\right)^2\right). \tag{2.27}$$

Replacing $\frac{U^A}{u_C \bar{Y}_A}$ by L_A yields

$$L_A = \frac{(\delta_C^A)^2}{100^2} + \frac{\delta_C^A}{100}\,. \tag{2.28}$$

To calculate the reduction of consumption equivalent to the social loss for region A, we solve for δ_C^A to obtain

$$\delta_C^A = 100\frac{-1+\sqrt{1+4(1-\gamma)L_A}}{2(1-\gamma)}\,. \tag{2.29}$$

The reduction of consumption equivalent to a certain welfare loss for region B can be obtained analogously. We use this transformation in the following subsections to make the welfare losses more tangible.

2.4.3 Evaluation of Policies in the Different Policy Games

In the following we examine the results of the simulations. The model calibration was explained in Section 2.4.1 and is summarized in Table 2.1. A summary of the results is given in Table 2.2.

Homogeneous Monetary Union

We begin with a comparison of the losses for the monetary and fiscal policy authorities in the symmetric case. The first columns of Table 2.2 show that the fiscal authorities of both regions face the highest region-specific policy losses under cooperation and in the scenario where monetary policy moves first. The lowest fiscal losses occur when fiscal policies have the greatest influence, i. e. under the scenarios of fiscal cooperation when fiscal policies move first and under fiscal cooperation in the simultaneous scenario. The explanation is simple: Fiscal policies aim at higher inflation and higher output than the central bank, which targets socially optimal levels. Due to the low relative weight on output stabilization the central bank reacts strongly to offset inflation deviating from the socially optimum level. Fiscal policies themselves engage in a trade-off between inflation and output when fixing their own policy decisions. An expansionary fiscal policy pushes output above the socially optimal level by granting subsidies in order to lower production costs. Thus it decreases inflation at the same time. Accordingly, output is higher than natural output and lower than the desired fiscal targets. Inflation is below the fiscal target levels and slightly below the social optimum. Note, however, that the central bank reacts strongly to the downward pressure of inflation with an expansionary monetary policy on account of the high weight on inflation in the target function.

The loss in the Nash scenario is similar to that of the two scenarios where fiscal policies move first.

In the scenarios where monetary policy takes lead (with or without coordination of fiscal policies), fiscal policies internalize the fact that the central bank cannot offset a fiscal policy that is too expansionary. Therefore, fiscal policies are less expansionary, and output and inflation deviate from the fiscal targets to a higher degree than in the previously analyzed scenarios. This implies higher losses for the fiscal policy authorities. The highest losses occur when policy makers cooperate and agree on the socially optimal targets: On average, the realized value for inflation is close to zero (but still dependent on stochastics) and output is at its lowest compared to the desired levels. It is, therefore, questionable whether overall cooperation aiming at socially optimal targets can be implemented in this setting.

Our assumption of a welfare-maximizing monetary policy means that the rankings of the central bank losses correspond to the rankings of the union-wide social losses. The social losses, in turn, can be transformed into welfare equivalent consumption reductions relative to the social optimum. Accordingly, we consider only the consumption losses of the private agents in the following.

Table 2.2: Baseline Model – Analysis of Welfare and Policy Losses

	Symmetric case					*Asymmetric case*				
	Calculated Policy Losses			Equivalent Consumption Reduction, %		Calculated Policy Losses			Equivalent Consumption Reduction, %	
Policy	L_{FA}	L_{FB}	L_M	CR_A	CR_B	L_{FA}	L_{FB}	L_M	CR_A	CR_B
Nash	21.909	21.909	0.119	0.012	0.012	23.824	51.163	0.363	0.024	0.065
	(0.033)	(0.033)	(0.000)	(0.000)	(0.000)	(0.019)	(0.054)	(0.000)	(0.000)	(0.000)
Stackelberg, fiscal leadership	21.911	21.911	0.116	0.012	0.012	23.803	51.170	0.350	0.024	0.062
	(0.033)	(0.033)	(0.000)	(0.000)	(0.000)	(0.019)	(0.054)	(0.000)	(0.000)	(0.000)
Stackelberg, mon. leadership	23.639	23.639	0.0160	0.002	0.002	33.439	85.522	0.004	0.000	0.001
	(0.032)	(0.032)	(0.000)	(0.000)	(0.000)	(0.016)	(0.055)	(0.000)	(0.000)	(0.000)
Cooperation	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
...region-specific policy losses	31.250	31.250	—	—	—	31.250	101.250	—	—	—
	(0.125)	(0.125)				(0.063)	(0.193)			
Fiscal coop., simultaneous	21.909	21.909	0.119	0.012	0.012	32.026	32.026	0.363	0.024	0.065
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.003)	(0.000)	(0.000)	(0.000)
...region-specific policy losses	21.909	21.909	—	—	—	23.825	51.162	—	—	—
	(0.033)	(0.033)				(0.019)	(0.054)			
Fiscal coop., fiscal leadership	21.645	21.646	0.111	0.011	0.011	32.466	32.466	0.413	0.022	0.087
	(0.010)	(0.010)	(0.000)	(0.000)	(0.000)	(0.042)	(0.042)	(0.001)	(0.000)	(0.000)
...region-specific policy losses	21.646	21.646	—	—	—	23.347	53.744	—	—	—
	(0.041)	(0.026)				(0.012)	(0.146)			
Fiscal coop., mon. leadership	31.241	31.241	0.000	0.000	0.000	52.225	52.225	0.000	0.000	0.000
	(0.377)	(0.377)	(0.000)	(0.000)	(0.000)	(0.434)	(0.434)	(0.000)	(0.000)	(0.000)
...region-specific policy losses	31.241	31.241	—	—	—	31.237	101.195	—	—	—
	(0.740)	(0.740)				(0.718)	(1.745)			

Remarks: L_{Fi} is fiscal loss in region i, L_M loss of the common central bank, all multiplied by 10^5. CR_i denotes welfare loss measured in terms of an equivalent permanent percent reduction in consumption in region i, relative to the pre-shock steady state. The numbers in parentheses denote standard deviations.

We find that the ranking of the scenarios is quite different in comparison with the (fiscal) policy makers' losses (see again Table 2.2). The first best can be attained in the cooperation scenario.[19] The consumption loss is also very low in both monetary leadership scenarios, i.e. when fiscal policies do not cooperate and when fiscal policies are coordinated. The highest social losses occur when fiscal policies are dominant in the sense of being Stackelberg leaders, and in the Nash scenario. In line with the explanation for the fiscal policy makers' losses, inflation and output levels are closest to the social optimum when monetary policy takes the lead (together, of course, with the joint cooperation case).

Heterogeneous Monetary Union

In our analysis of a heterogeneous monetary union we assume that the fiscal policy of region A follows the same strategy as in the homogeneous case, whereas the fiscal policy of region B targets higher levels of both inflation and output. Furthermore, we assume that region B is smaller than region A and is characterized by a slightly higher degree of price-stickiness. The exact parameter values for region A are again depicted in the second column of Table 2.1, while the "alternative" parameter values for region B are summarized in the third column of this table. Results for the heterogeneous case are shown in columns seven to eleven of Table 2.2.

Beginning with the losses for region A, we find that the values of the fiscal policy maker's losses are much higher for all scenarios in the heterogeneous case except one: The cooperation scenario corresponds to the homogeneous case by definition, as all policy makers agree on the socially optimal targets. The ranking of the scenarios with respect to the region-specific fiscal policy makers' losses is similar to that in the homogeneous case: The highest losses occur when monetary policy has the greatest influence (monetary leadership scenarios), the smallest losses occur in the scenarios in which fiscal policies have the greatest influence (fiscal cooperation when fiscal policy takes leadership, fiscal cooperation and simultaneous decision-making, and fiscal leadership when monetary policy is uncoordinated), and in the Nash scenario. The fiscal policy maker again faces the highest loss in the joint cooperation scenario. We observe almost the same ranking for region B, but the losses are higher compared to region A.

We find that the losses of the common central bank and, hence, also the consumption losses of the private agents show also a similar ranking as in the homogeneous mone-

[19] The (monetary) policy loss is slightly larger than zero because of the shock in our simulation.

tary union: The lowest losses are attained when monetary policy moves first or when all policy makers agree on the socially optimal targets (=first best). The highest losses occur when fiscal policies moves first (uncoordinated and coordinated) and when fiscal policies are coordinated and monetary and fiscal policy decisions take place simultaneously. This result seems, at first glance, to be contrary to the findings of Lombardo and Sutherland (2004), who state that fiscal cooperation is welfare-improving. But a closer look reveals that our calibration of a unit elasticity of substitution between domestic and foreign goods also implies in Lombardo and Sutherland (2004), according to their Proposition 1, that fiscal cooperation is no longer welfare-improving.[20]

The welfare-equivalent consumption reductions under Nash, fiscal leadership, and the two fiscal cooperation scenarios with simultaneous actions or with fiscal leadership are about three times larger in the (smaller) region B. Also, the equivalent consumption reductions are relatively higher in the heterogeneous case compared to the homogeneous case, by about 50 percent for region A and a factor of above four for region B. This implies that a model of a homogeneous monetary union that does not properly take into account heterogeneities possibly underestimates the welfare effects of certain policies. This finding also suggests that homogeneity is a desirable feature of the currency area for all policy makers (fiscal and monetary authorities) and the private agents.

2.4.4 Sensitivity Analysis

Are the results of the previous section robust to changes in the structural parameters of the model? To examine this, we vary the structural parameters within plausible ranges. In Figure 2.4 we plot the parameter variations that show the highest sensitivity of results. The corresponding parameters are the elasticity of marginal disutility of labor (emdl), price rigidity ϕ_i, and the elasticity of substitution θ. We plot their effects on fiscal policy makers' losses and social welfare, which is equivalent to the central bank loss for both the symmetric and the asymmetric case.[21] To see how the figure should be read, we focus on column five. It shows the monetary losses in the asymmetric mone-

[20] Note also that Lombardo and Sutherland (2004) features government consumption in the utility function.

[21] In the figures we use the following abbreviations to save space: For the policy scenarios, Nash = Nash, Coop = cooperation, FCoop = fiscal cooperation, FLead = fiscal leadership, MLead = monetary leadership, FCFL = fiscal cooperation with fiscal leadership, FCML = fiscal cooperation with monetary leadership. The labels on the x-axis denote emdl = elasticity of marginal disutility of labor, Φ = Calvo parameter, i.e. the percentage of firms that cannot adjust their prices, and θ = elasticity of substitution between different goods produced in the same region.

tary union for changes in the three parameters elasticity of marginal disutility of labor, price stickiness and substitution elasticity. Given our assumption of monetary policy, the monetary loss equals the socially optimal loss. The dashed green line in rows 1, 3 and 5 shows that the cooperation scenario leads to zero welfare losses for the whole range of parameter values. In rows 2, 4 and 6 we see that the two monetary leadership scenarios also result in small welfare losses. The welfare ranking of scenarios is robust across changes in the parameter values. In the following we take a more detailed view on each parameter variation.

Variation of the Elasticity of Marginal Disutility of Labor

We vary the elasticity of marginal disutility of labor (emdl) between zero and one, where the lower bound is given in Blanchard and Fischer (1989), while the upper bound is often used in New Keynesian models, see e. g. Galí and Monacelli (2005a). The effects of these variations on the policy losses in the three simultaneous scenarios are depicted in the first row of Figure 2.4, while the second row shows the effects in the four sequential scenarios.

Increasing elasticity of marginal disutility of labor leads to higher central bank losses. This result is obvious as, given the other parameters, the same outcome is produced at higher cost, meaning that the same effort in the production of goods leads now to a higher reduction of utility than before.

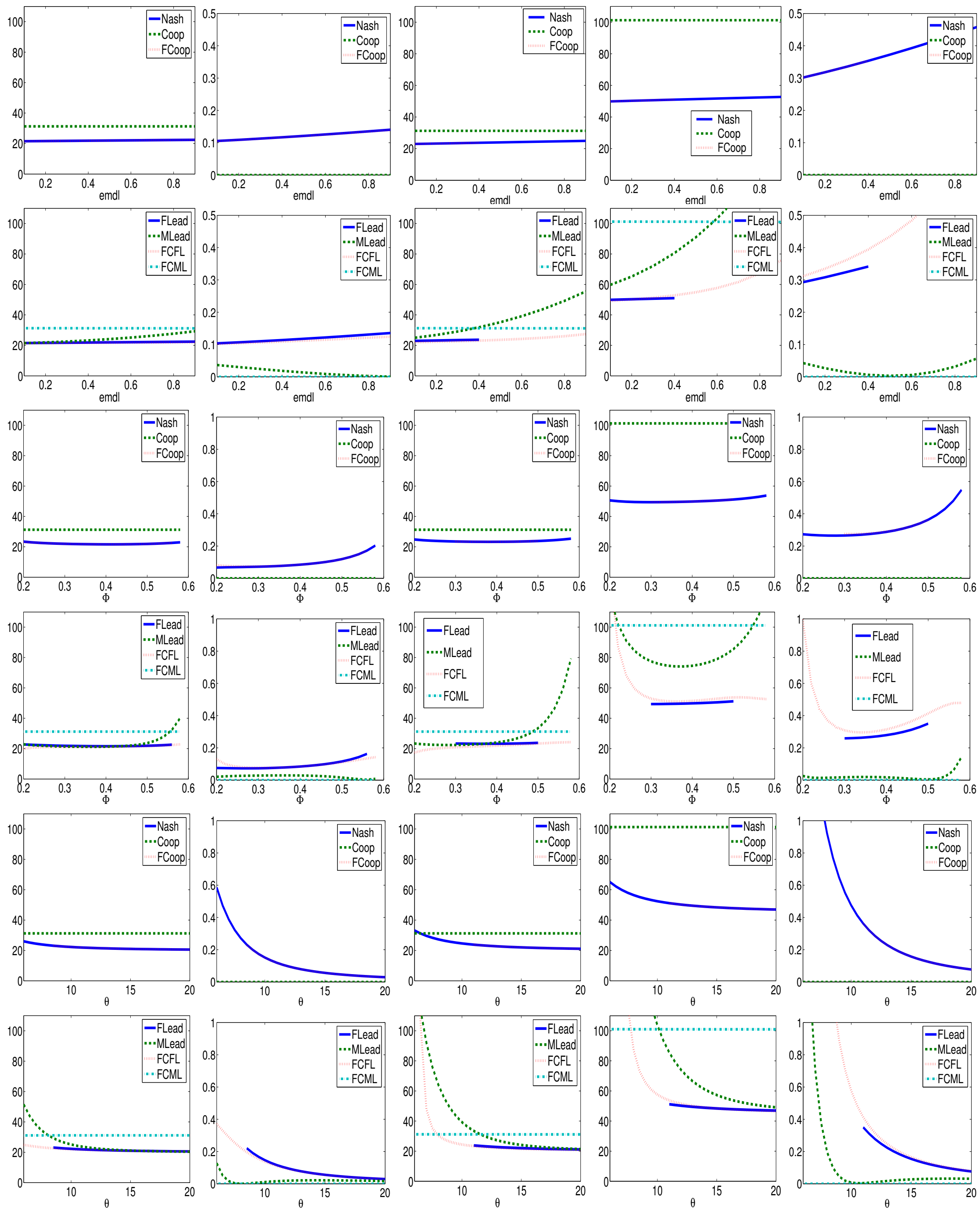
Referring to the homogeneous case, we see that the rankings for both the fiscal authorities' losses and the central bank losses are stable: fiscal policies suffer from the smallest losses in the Nash scenario and if they obtain fiscal leadership, as in comparison with the other scenarios they are better able to pursue their inflation and output targets (above the socially optimal levels). The central bank's welfare function shows the smallest losses in the joint cooperation case (which determines the first best) and in the scenario where monetary policy takes leadership. In the latter scenario, the fiscal policies are restrained, as too expansionary a fiscal policy would lead to low inflation, which will not be corrected by the central bank afterwards. Therefore, monetary leadership has a disciplining effect on supply-side-oriented fiscal policies. The fact that joint cooperation leads to the first best from a welfare perspective comes as no surprise as all policy makers agree upon the socially optimal targets, as mentioned in the previous section.

In the heterogeneous case, the losses are higher for the fiscal policies of both regions,

Figure 2.4: Calculated Region-Specific Policy Losses for Parameter Variations in Region A and B

Homogeneous mon. union | Heterogeneous monetary union

$L_{FA} = L_{FB}$ | L_M | L_{FA} | L_{FB} | L_M



the one with the more conservative and the one with the more aggressive targets, and also for the central bank. However, the rankings seem to be robust with two exceptions: (i) When monetary policy moves first fiscal losses are strongly increasing for higher values of the elasticity of disutility of labor. (ii) The losses in the fiscal cooperation fiscal leadership cases increase a lot at a value of 0.4, which may be an indication that there is no equilibrium to which rational inflation expectations could converge. It would be interesting to take up this point in further research.

Variation of Price Rigidity

The third and fourth rows of Figure 2.4 examine the effect of varying price rigidities on fiscal and monetary losses. The figure shows that the ranking of the scenarios is stable in the homogeneous and heterogeneous case for almost the whole parameter set, and it is in line with the results of Table 2.2: Fiscal policies incur the smallest loss under fiscal leadership, whereas monetary policy suffers from the smallest losses when it takes leadership and, of course, under the joint cooperation scenario. Again, the fiscal cooperation fiscal leadership scenario leads to dramatically increasing losses for more rigid prices, a factor that calls for an analysis in future research.[22]

Variation of the Elasticity of Substitution of Consumption Goods

In the fifth and sixth rows of Figure 2.4 we consider the effect of changes from the elasticity of substitution of consumption goods, θ, on the losses over the range discussed by Obstfeld and Rogoff (2001). The figure confirms one intuitive result, i.e. that an increasing θ leads to smaller fiscal policy and welfare losses: higher substitutability between goods implies fewer distortions from monopoly power. There is again one interesting exception. For a relatively small value of θ below 10 the losses become very big, which again may conceivably induce indeterminacy of equilibria.

Summary of the Findings

For all parameter variations over the ranges used in the standard literature (see our calibration), we find that the rankings of the different scenarios illustrated by Table 2.2 are

[22] The variations of the intertemporal discount factor η, which determines the importance of "pseudo-future" periods relative to the present period in the producer-consumers price-setting behavior, show almost the same results as those indicated for variations of the price rigidity parameter. We, therefore, abstain from depicting and discussing the figures for η.

relatively robust. The sensitivity analysis has also confirmed that the losses in a heterogeneous monetary union tend to be higher. From the perspective of welfare maximization, joint cooperation and monetary leadership are the best-performing scenarios.

Looking at the general picture, especially the equivalent consumption reductions reported in able 2.2 are small in absolute value, even in the heterogeneous monetary union. Therefore, one might be tempted to argue that heterogeneities are unimportant. We think this is misleading, for two reasons. First, we consider only four sources of heterogeneities: regional size, home bias, price rigidities and differences in policy targets. These alone already increase the consumption equivalent losses by a factor of up to 8. If more heterogeneities were to be incorporated, the drawbacks of the relatively worse performing scenarios might become non-negligible. Second, the current economic downturn highlights many potential risks to the European monetary union. All of these risks are related to country specific problems, so that the issue of heterogeneities in a monetary union receives renewed attention.

2.5 Conclusion

In this paper we have examined the interactions of fiscal and monetary policies in a monetary union. One main focus was to derive a theoretical model that allows for capturing heterogeneities among the different countries participating in a monetary union, and for analyzing strategic interactions of fiscal and monetary authorities. Why do heterogeneities matter? The answer is relatively simple. By adopting the Euro, the participating countries abstain from a monetary policy of their own and fiscal policy remains the only instrument for pursuing region-specific goals and stabilizing region-specific shocks. The common central bank has to implement a monetary policy that is most appropriate for the whole monetary union, and cannot respond to idiosyncratic shocks and country-specific political targets. This makes the role of fiscal policies more important and leaves room for strategic behavior in achieving national goals.

To examine these heterogeneities we have enhanced the model of Dixit and Lambertini (2003*b*). From the microfoundation we have established that a region-specific productivity shock and terms of trade have an impact on regional output. In Section 2.3 we introduced different possible scenarios for strategic interactions between fiscal and monetary policies. In this context we assumed that fiscal policies deviate from optimizing regional welfare, aiming instead at higher inflation and output compared to the union-wide central bank. By contrast, monetary policy is assumed to maximize union-wide welfare.

We have used simulations to evaluate the different scenarios of strategic behavior for supply-side fiscal policies in line with the micro-model. These aim at granting subsidies to increase output financed by per-capita taxes. We have thus considered a heterogeneous monetary union comprising two different regions: a “conservative region” and a “catch-up” region. We have assumed that the desired inflation and output targets of the “conservative region” are relatively closer to the social optimum.

To evaluate the different policy games, we have used a calibration of our micro-model drawing upon the parameters from the standard economic literature. We have shown that the losses of fiscal policies are relatively small in the Nash scenario, in the fiscal leadership scenario (for both cooperation of fiscal policies and independently acting fiscal policies), and when fiscal policies cooperate and all policy makers move simultaneously. In these scenarios, fiscal policies achieve an output level closest to their preferred levels, whereas inflation is stabilized close to the socially optimal level by the common central bank.

The losses of monetary policy, which correspond to the welfare losses of the private agents, are lowest when monetary policy moves first. The first-best situation is attained when all policy makers agree upon the socially optimal levels. But as the central bank and fiscal policy makers consider different scenarios optimal such, an agreement appears to be unrealistic on a voluntary basis.

In the EMU, fiscal policies appear primarily to track national interests. However, the analysis has shown that fiscal policies in a heterogeneous monetary union can contribute to high welfare losses. From a welfare perspective, monetary leadership or cooperation would then be a desirable scenario for both types of fiscal policy.

To summarize the main findings, we state that if the authorities’ preferences do not coincide, or are at least relatively far apart, worse outcomes are likely to occur. In such a case, designing the institutions so that monetary policy plays a lead role generates the smallest losses for the agents living in both regions, even with existing heterogeneities.

The European Central Bank aggressively pursues the price-stability goal, meaning that the inflation rate should not exceed 2%. Accordingly, it appears to act as a first mover, which is beneficial for welfare. At the same time, fiscal policies are restricted in their actions by the Stability and Growth Pact, which leaves less room for pursuing excessive fiscal targets and implies a reduction of the trade-offs caused by strategic behavior.

Recent experience, however, has shown that in bad times meeting the stability criteria may not be a very credible option for fiscal policies, especially, when the culprits judge their own sanctions, as it has happened in the European Union. Therefore, reducing heterogeneities and bringing fiscal policies' targets closer to the socially optimal levels is an essential task in achieving a longer-term stability guarantee for the EMU.

3 The Macroeconomics of Real Estate

with Harald Uhlig

Is it possible to explain the house price to GDP ratio and the house price to stock price ratio as being generally constant, deviating from its respective mean only because of shocks to productivity. We build a two-sector RBC model for residental and non-residental capital with adjustment costs to capital in both sectors. We show that an anticipated future shock to trend productivity in the non-residental sector leads to a large increase in house prices in the present. We use this property of the model to explain the current house price behavior in the U. S., the U. K., Japan and Germany.

3.1 Introduction

Many researchers and practitioners are puzzled by the developments in the real estate market prior to the downturn in 2006. While countries such as the U.S., Spain or the U.K. experienced high growth rates in real estate prices, real estate prices did not grow at all or even fell in countries like Japan, Germany and Switzerland. This may explain the growing interest of researchers in understanding the real estate market in the recent years, even before the beginning of the current recession. While European Central Bank (2003) and International Monetary Fund (2004) report some stylized facts for real estate prices – they co-move with GDP, but are lagging, more volatile and do have longer cycles of more than ten years – there is rather little understanding about the driving forces in the real estate market. This is the starting point of this paper. We lay out our theory to simultaneously explain movements in GDP, house prices and stock prices. We assume that all three variables feature the same balanced growth path. But temporary shocks may lead to deviations in the house price to GDP and the house price to stock price ratio. The main point of this paper is to show how expectations about the prospects of the economy can explain an immediate rise in house prices relative to stocks and GDP. We make this point by building a two-sector RBC model for non-residential capital and housing capital. Both sectors are subject to concave adjustment costs, but only the real estate sector features a constant factor of production. Because of the latter assumption, the relative price of real estate has a positive trend. From a modeling perspective, this paper is related to Piazzesi et al. (2007), Benhabib, Rogerson, and Wright (1991) and Greenwood and Hercowitz (1991). While all papers have a consumption side that is comparable to our model, the first and the second paper lack an explicit production function. The second and the third paper answer a different question: They analyze the implications of non-market activities by households. Nonetheless, the modeling strategy is similar in its approach.

Of course, house price movements have already been addressed in many papers. A survey article of the relevant literature can be found in Leung (2004). For this reason we will only briefly mention some of the papers here. Case, Glaeser, and Parker (2000), Campbell and Cocco (2005) and Iacoviello (2005) all explore the connections between real estate and consumption and their possible implications for economic (fiscal or monetary) policy. The models presented in Iacoviello (2005), Ortalo-Magné and Rady (2002) as well as Jin and Zeng (2004) and Yang (2005) all feature household heterogeneity with respect to borrowing constraints, homeownership or age. Lustig and Van Nieuwerburgh (2005) investigate the effect of changing house prices on the housing wealth to human wealth ratio in a model of housing collateral.

The effect of inflation on house prices – either as an effective tax subsidy to owner occupied housing or through money illusion – is analyzed in a series of papers by Poterba (1984, 1991, 1992) and, more recently, Brunnermeier and Julliard (2008). Piazzesi and Schneider (2008) build a heterogeneous agent model in which next to the tax channel heterogeneous inflation expectations increase the volume of credit and thus the price of the collateral.

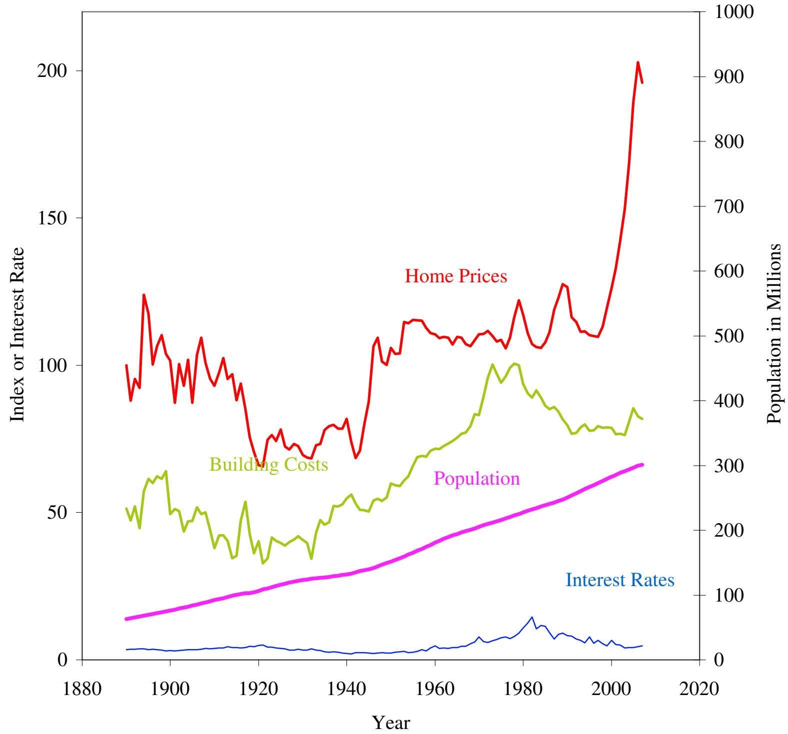
Our approach differs from most of the literature in two respects. First, we want to explain house price movements in response to changes in the macroeconomic environment. Second, instead of resorting to individual credit constraints or household heterogeneities, we consider only shocks to current and future productivity as driving forces.

The paper is structured as follows. Section 3.2 summarizes important empirical findings. Section 3.3 presents our main idea in a simplified endowment economy. An elaborated production economy, its solution and its results are dealt with in Section 3.4, followed by the conclusion in Section 3.5.

3.2 Some Facts

In the second edition of his bestseller book on irrational exuberance, Shiller (2005) especially focuses on the development of house prices. In one of his graphs, he shows that the development in house prices over the last 120 years can be explained neither by building costs nor by population nor by interest rates. We replicate this graph in Figure 3.1. So he assumes a sizeable degree of irrationalism in the market for real estate.

Figure 3.1: House Prices development in the long run: Figure 2.1 from Shiller (2005)

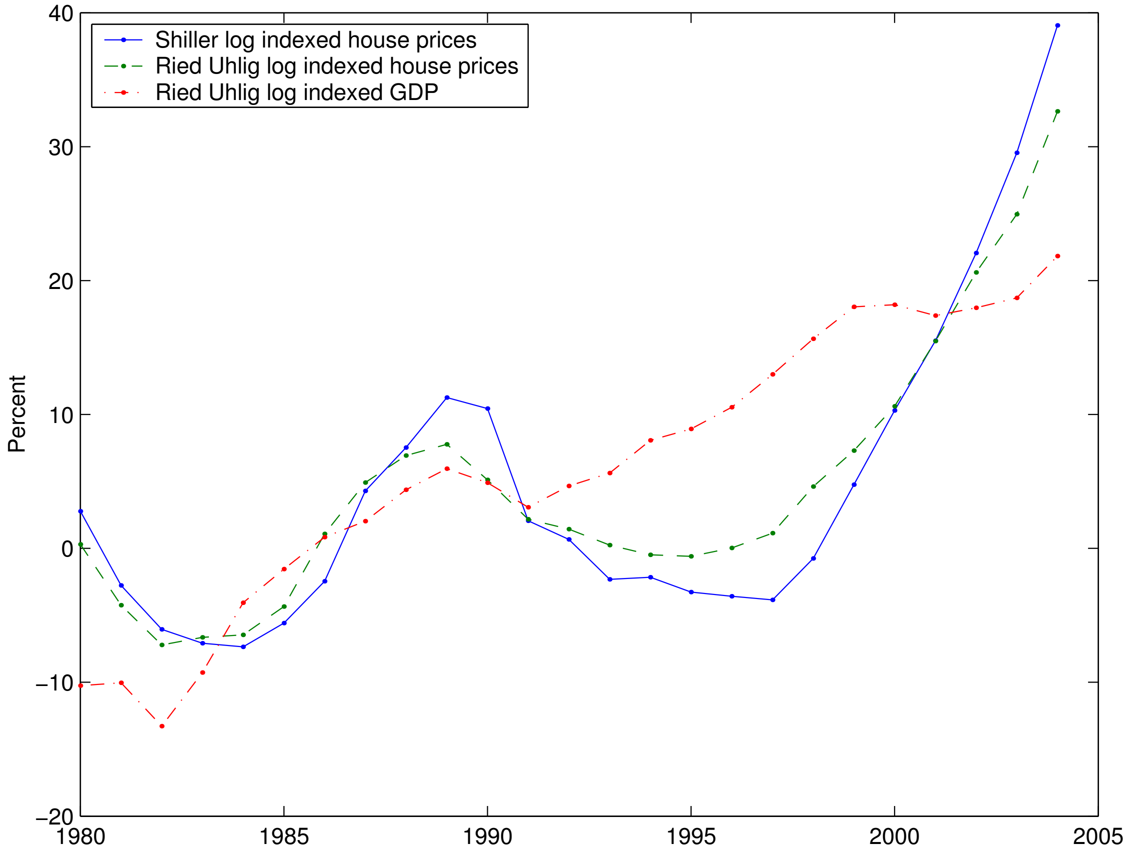


But is this really the only valid conclusion? In Figure 3.2 we look at the period from 1980 onwards only and compare the data Shiller kindly provides on his webpage[1] to ours.[2] We find it to be very similar. Furthermore, we compare it to real GDP and find that the evolution of house prices does not look that odd once we compare it to the evolution of real GDP. So the task seems be much simpler: Instead of attempting to explain the dramatic increase in house prices, the question is only why house prices increased

[1] See `http://www.econ.yale.edu/ shiller/books.htm`.

[2] We have chosen to log the data and to index it so that the mean between 1908 and 1995 equals zero.

Figure 3.2: Comparison of Shiller data with our data, U.S. log indexed house prices from 1980 onwards: not too many differences, and less dramatic



relative to GDP over the last years before the downturn. To put this into context, we also look at the house price to stock price ratio in order to see whether or not there are similarities.

Figures 3.3 to 3.5 present the evolution of (log) real GDP, the ratio of (log) real house prices (HP) to real GDP and the ratio of (log) real house prices to (log) real stock prices (SP) for the U. S., the U. K., Japan and Germany. For GDP, we use seasonally adjusted real per capita GDP. The NYSE composite index and OFHEO house prices are used for U.S. prices. For U.K. prices, we use the FTSE 30 ordinary share index and the nationwide house price index of the Nationwide Building Society. The Nikkei 225 index and nationwide consumer prices for "Housing, Water, Electricity, Gas and Other Fuels" are used for Japan, whereas German prices are the DAX 30 index and construction prices. All series are deflated using the CPI and indexed such that the average of 1980:I - 1995:IV equals zero. For more details, see appendix Section 5.2.1. We see that the house price to GDP ratio seems to be relatively stationary. It is most volatile in the U. K. and in the U. S., but much less volatile than the house price to stock price ratio. In the last ten years houses have become more expensive in the U. K. and Japan, but less expensive in the U. S. and Germany relative to the own country's stock market.

Figure 3.3: GDP, House Prices to GDP Ratio and Stock Prices to GDP Ratio for the U. S.

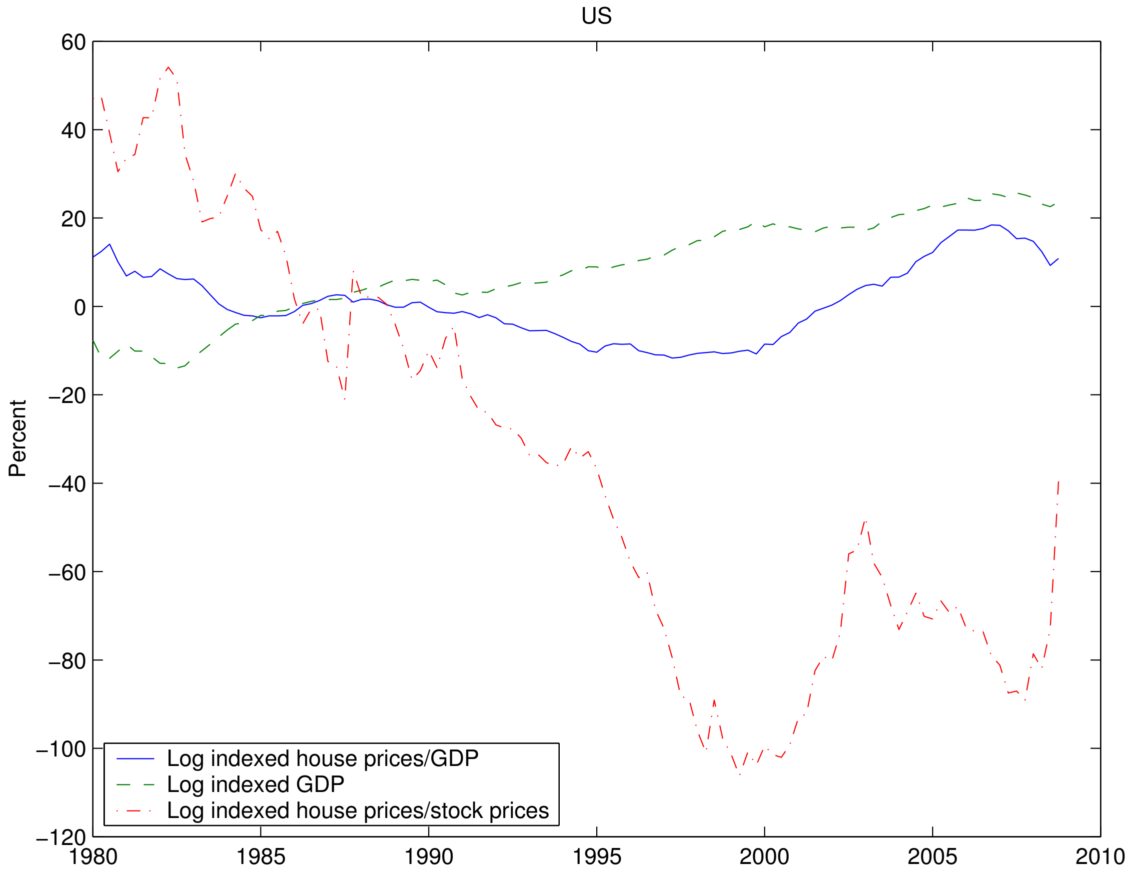


Figure 3.4: GDP, House Prices to GDP Ratio and Stock Prices to GDP Ratio for the U. K., Japan and Germany

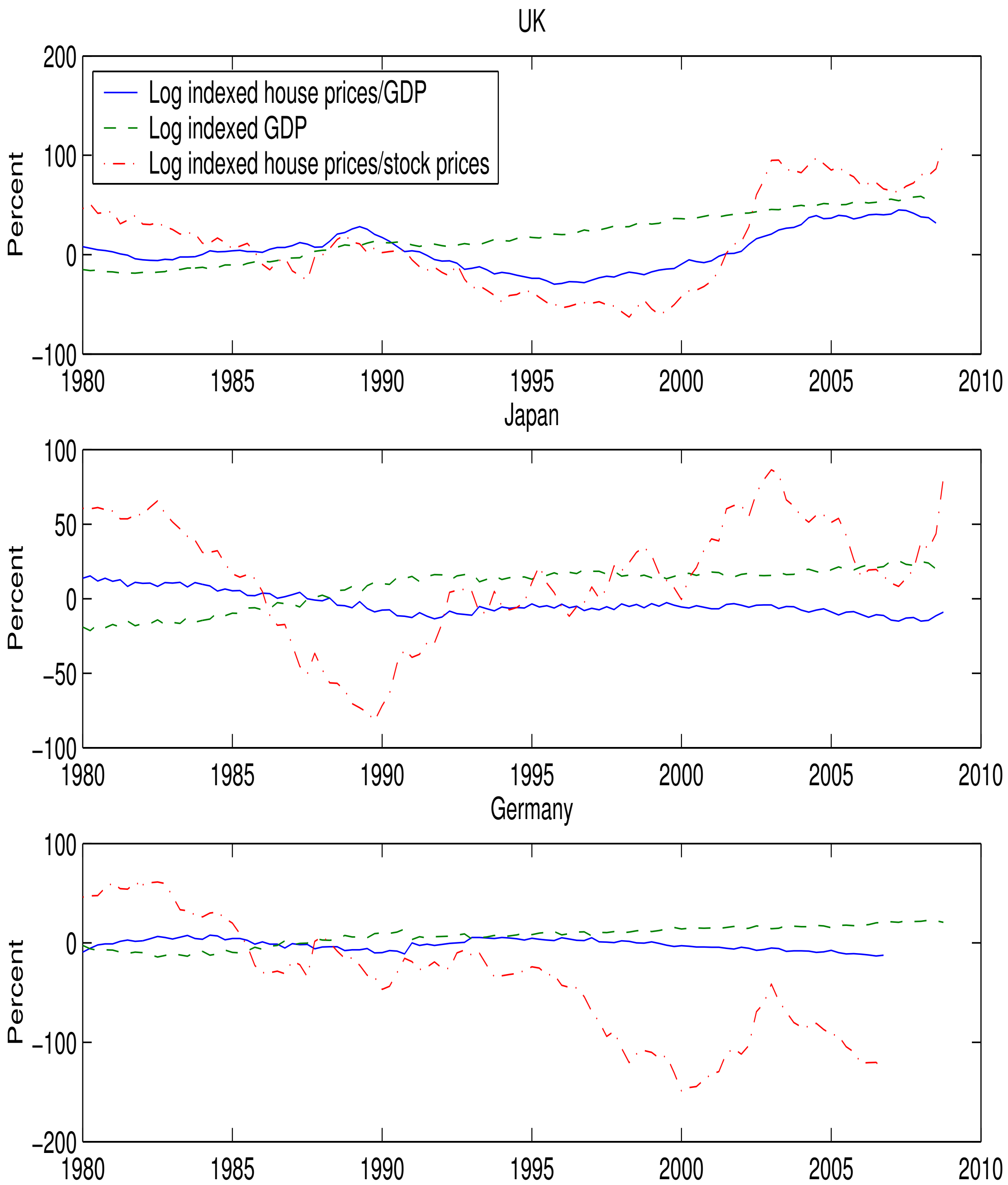
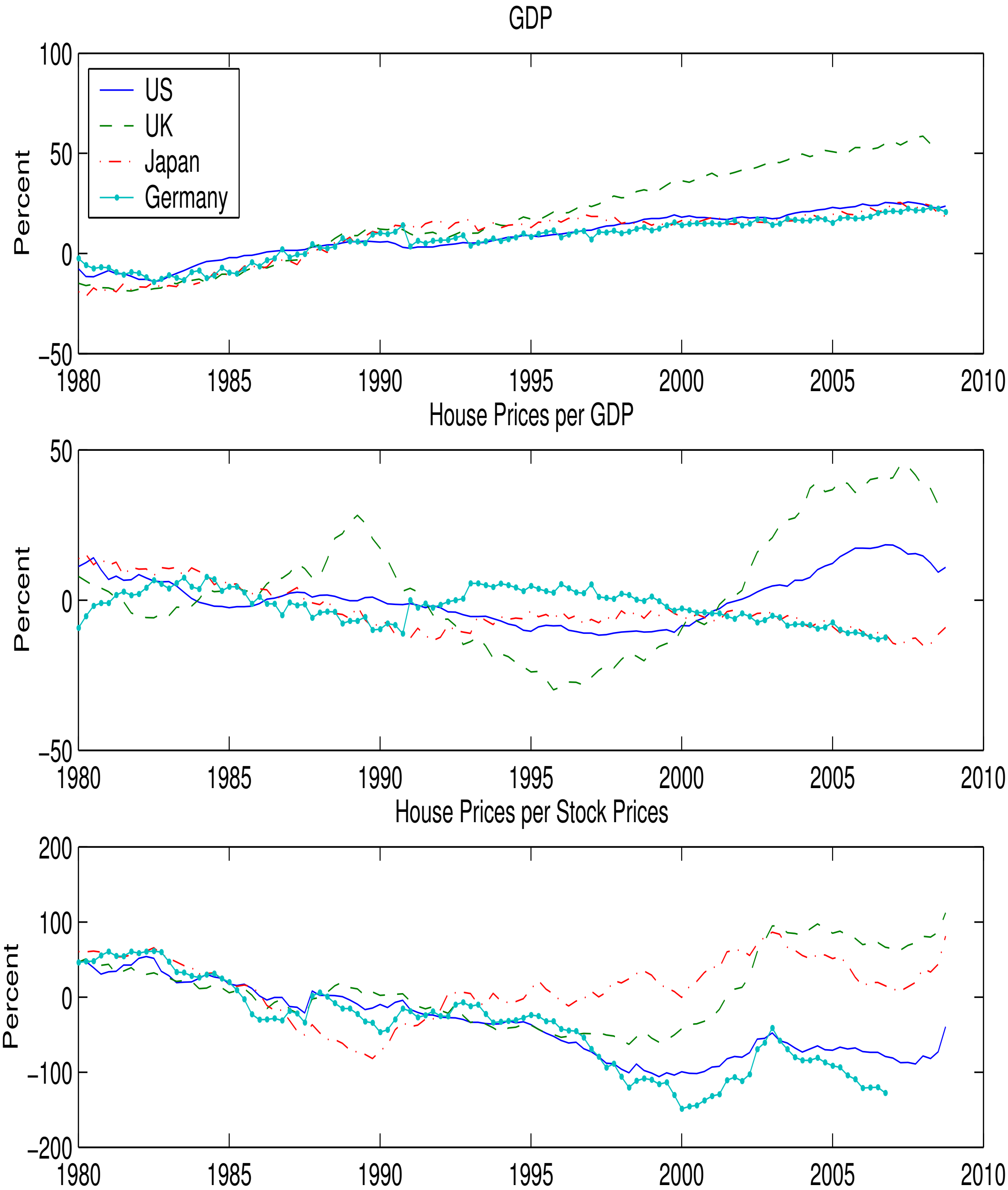


Figure 3.5: Comparison between the U.S., the U.K., Japan and Germany: GDP, House Prices to GDP and House Prices to Stock Prices Ratio



3.3 An Endowment Economy

We start with a stripped down endowment economy to preview the gist of the main model laid out in the following section. Time is discrete. There are two kinds of goods: a consumption good and housing services. They are produced by "Lucas trees": the "consumption tree" produces z_t units of consumption per tree, while the "housing tree" produces s_t units of housing services. Let a_t and b_t denote the ownership of consumption trees and housing trees by the household. A representative agent has preferences of the form

$$\max_{c_t, s_t, a_t, b_t} E_t \sum_{t=0}^{\infty} \beta^t (\theta \log c_t + (1-\theta) \log s_t) \tag{3.1}$$

for consumption good c_t and housing services s_t. The housing tree may be used to a certain percentage, as determined by $s_t \in [0, 1]$. Consumption equals the production of the consumption tree: $c_t = z_t$. Households trade in the ownership a_t of the consumption tree and the ownership b_t of the housing tree, the latter resulting in one unit of housing services. The according prices are p_t and q_t, respectively. Given the relative price of housing services r_t, the budget constraint of the household is:

$$c_t + r_t s_t + p_t a_t + q_t b_t = (p_t + z_t) a_{t-1} + (q_t + r_t) b_{t-1} . \tag{3.2}$$

Market clearing requires $a_t = b_t = s_t = 1$ and $c_t = z_t$. For the production function this means that $c_t = 1$ as well as $s_t = 1$. Defining output as the sum of consumption and housing services,

$$y_t \equiv c_t + r_t s_t , \tag{3.3}$$

in equilibrium the following equation holds:

$$c_t + r_t = z_t + r_t = y_t . \tag{3.4}$$

Optimal allocation results in the following expenditure shares:

$$c_t = \theta y_t \qquad \text{and} \qquad r_t = (1-\theta) y_t . \tag{3.5}$$

For the rental rate r_t, this can be transformed to show the relation to productivity:

$$r_t = \frac{1-\theta}{\theta} z_t . \tag{3.6}$$

As further first-order necessary conditions one obtains two asset pricing equations for stocks and real estate:

$$p_t = \beta E_t[\frac{c_t}{c_{t+1}}(z_{t+1} + p_{t+1})] \tag{3.7}$$

$$q_t = \beta E_t[\frac{c_t}{c_{t+1}}(r_{t+1} + q_{t+1})]. \tag{3.8}$$

The fundamental solution to Equation (3.7) is

$$\frac{p_t}{z_t} = \frac{\beta}{1-\beta} \tag{3.9}$$

which can be transformed by using $z_t = \theta y_t$ to give a stock price to GDP ratio

$$\frac{p_t}{y_t} = \theta \frac{\beta}{1-\beta}. \tag{3.10}$$

Equivalently, the house price to GDP ratio is given by

$$\frac{q_t}{y_t} = (1-\theta)\frac{\beta}{1-\beta} \tag{3.11}$$

and the house price to stock price ratio

$$\frac{q_t}{p_t} = \frac{1-\theta}{\theta}. \tag{3.12}$$

In this setup, movements in GDP should be directly reflected in movements in stock and real estate prices, where the size of the movement in prices depends on the size of the utility parameter θ. If this parameter was modelled as a random variable, changes in current GDP would have time-variant (positive) effects on stock and real estate prices.

3.3.1 Balanced Growth Path

If real estate is constant, but the exogenous income source is assumed to grow according to

$$z_t = \gamma^t, \gamma > 1, \tag{3.13}$$

the price of housing services r_t is growing with the exogenous income source z_t. According to equation (3.6), it is

$$r_t = \frac{1-\theta}{\theta}\frac{c_t}{s_t} = \frac{1-\theta}{\theta}\gamma^t \,. \tag{3.14}$$

If z_t is interpreted as productivity, house prices grow proportionately with productivity.

Hence, this stylized endowment economy features constant house price to GDP and house price to stock price ratios. Given a constant stock of real estate, trend growth in productivity and, thus, GDP translate into trend growth of the same size in house prices. We use this building blocks for the more elaborated production economy of the next section. There, prices and productivity will be specified with more detail.

3.4 A Production Economy

In this model, there are two sectors: Consumption goods are produced using consumption good capital and labor. Real estate is produced using real estate capital and constant land. Both types of capital are subject to adjustment costs. The model bears similarity to Uhlig (2004), with the extension for two sectors.

The representative agent maximizes utility over consumption c_t, housing services s_t and leisure $1-n_t$ as follows:

$$\max_{c_t,s_t,n_t,x_{c,t},x_{s,t},k_{c,t},k_{s,t}} E_t \sum_{t=0}^{\infty} \beta^t \left(u(c_t, z_{s,t}, s_t) - An_t\right) \,, \tag{3.15}$$

where $z_{s,t}$ denotes a preference shock and n_t denotes labor, subject to the budget constraint, two production functions and two equations for capital accumulation. The utility function $u(\cdot)$ is assumed to be additively separable[3] and of the form

$$u(c_t, z_{s,t}, s_t) = \theta \log c_t + (1-\theta) z_{s,t} \log s_t. \tag{3.16}$$

The agent is endowed with one unit of time per period and L units of land. The budget constraint is given by

$$c_t + x_{c,t} + x_{s,t} = y_{c,t} \,, \tag{3.17}$$

where $x_{c,t}$ is investment in the consumption good production $y_{c,t}$ and $x_{s,t}$ is invest-

[3] In a similar model with two different growth patterns, Greenwood, Hercowitz, and Krusell (1997) report that in the class of constant elasticity of substitution models, balanced growth is only permitted in the simple case like the one considered here.

ment in housing. These constructs are normalized so that every real quantity is measured in units of consumption.

The production technology in the consumption good sector is given by the Cobb-Douglas function in consumption good capital and productive labor $z_{c,t}n_t$,

$$y_{c,t} = k_{c,t-1}^{\alpha_c}(z_{c,t}n_t)^{1-\alpha_c} \ . \tag{3.18}$$

Investment in housing increases real estate production and also housing services. For reasons of simplicity we assume real estate production to be equal to the service flow it generates, hence, the words *housing services* and *real estate production* are used interchangeably. Real estate production has the Cobb-Douglas functional form in the arguments real estate capital and constant stock of land,[4]

$$s_t = y_{s,t} = k_{s,t-1}^{\alpha_s}L^{1-\alpha_s} \ . \tag{3.19}$$

In this setting, the agent builds real estate by investing in $x_{s,t}$, both measured in units of consumption, out of the income received from producing output in the consumption good industry $y_{c,t}$. Once real estate is built, it provides housing services s_t that do not have to be paid for additionally.[5] Note that there is no such thing as “income" from owning a house, as it is owner-occupied. Instead, the house provides utility through housing services. One could imagine an alternative model in which the representative household rents a house or flat from real estate firms. These firms work in a competitive market with zero profits and pay the marginal product to each production factor. As both production factors are owned by the representative household, this setting renders equivalent results. Capital accumulation in both sectors is subject to a concave adjustment cost function g to be specified later:

$$k_{i,t} = (1-\delta_i)k_{i,t-1} + g_i\left(\frac{x_{i,t}}{k_{i,t-1}}\right)k_{i,t-1}, \quad i \in \{c,s\}. \tag{3.20}$$

The exogenous process for housing preferences $z_{s,t}$ is assumed to be AR(1), while productivity $z_{c,t}$ is a unit root process with stochastic trend growth μ_t and an additional

[4]The fixed factor in real estate production can also be excluded by setting $\alpha_s = 1$.
[5]“Selling" real estate can be done by disinvesting in $x_{s,t}$.

shock:

$$\log z_{s,t} = \rho_s \log z_{s,t-1} + \epsilon_{s,t} \tag{3.21}$$

$$\log z_{c,t} = \log \mu_t + \log z_{c,t-1} + \epsilon_{c,t}, \text{ where} \tag{3.22}$$

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + (1-\rho_\mu)\log \bar{\mu} + \nu_t. \tag{3.23}$$

While the shock $\epsilon_{c,t}$ influences the level of productivity, ν_t influences the level of trend productivity and, hence, the growth rate of productivity. Below, we will focus on the effects of both shocks. Special attention will be put to the question of what happens to the growth rate of productivity, ν_{t+i} after an anticipated future shock. Note that Equation (3.23) incorporates long-run risk into the analysis. This has recently been emphasized in particular by Bansal and Yaron (2004) as well as Hansen, Heaton, and Li (2005) and Hansen and Scheinkman (2006). It would be interesting to study an extension of the model to include Epstein-Zin preferences, as done by Bansal and Yaron (2004) or Piazzesi and Schneider (2008), the latter paper even focussing on the real estate market. We have not done so here to keep the analysis as tractable as possible. With this, the model is closed. We are solving for the social planner solution. An equilibrium is given by the following definition.

Definition 3. *Given initial endowments $k_{c,-1}$ and $k_{s,-1}$, an equilibrium is an allocation $\{c_t, s_t, n_t, x_{c,t}, x_{s,t}, y_{c,t}, k_{c,t}, k_{s,t}\}_{t=0}^{\infty}$ such that markets clear and the allocation solves each household's problem.*

3.4.1 Analysis

The social planning problem stated above can be decentralized by a sector of competitive firms for consumption good production and for real estate production, respectively, as well as a household sector. The maximization problems of the two production sectors – both denoted in units of the consumption good – are

$$\max_{k_{c,t-1}, n_t} y_{c,t} - d_{c,t} k_{c,t-1} - w_t n_t \tag{3.24}$$

$$\max_{k_{s,t-1}, L} p_t y_{s,t} - d_{s,t} k_{s,t-1} - d_{L,t} L, \tag{3.25}$$

subject to the production function given in Equation (3.18) and (3.19), respectively. Here p_t is the price of real estate in terms of the consumption good. The wage w_t, the two dividends or rental rates on capital $d_{c,t}$ and $d_{s,t}$ as well as the land dividend $d_{L,t}$ are measured in units of the consumption good and taken as given by the firms. The representative household again maximizes the utility function given in Equations 3.15 and 3.16, subject to the capital accumulation Equations (3.20) and the budget constraint

$$c_t + k_{c,t} + p_t k_{s,t} = w_t n_t + d_{c,t} k_{c,t-1} + d_{s,t} k_{s,t-1} + d_{L,t} L_{,t}. \tag{3.26}$$

Maximization of firm profits results in four marginal products (wage rate and three dividends). Together with appropriate definitions of returns to investments in capital,[6] we have

$$d_{c,t} = \alpha_c \frac{y_{c,t}}{k_{c,t-1}}, \tag{3.27}$$

$$w_t = (1-\alpha_c)\frac{y_{c,t}}{n_t} \tag{3.28}$$

$$d_{s,t} = \alpha_s p_t \frac{y_{s,t}}{k_{s,t-1}}, \tag{3.29}$$

$$d_{L,t} = (1-\alpha_s) p_t \frac{y_{s,t}}{L}, \tag{3.30}$$

$$R_{i,t+1} = g_i'\left(\frac{x_{i,t}}{k_{i,t-1}}\right)\left(d_{i,t+1} + \frac{1-\delta_i + g_i\left(\frac{x_{i,t+1}}{k_{i,t}}\right)}{g_i'\left(\frac{x_{i,t+1}}{k_{i,t}}\right)} - \frac{x_{i,t+1}}{k_{i,t}}\right), i \in \{c,s\}. \tag{3.31}$$

All of these variables are measured in units of the consumption good. The marginal products of real estate production are translated into units of the consumption good by using the relative price

$$p_t = \frac{(1-\theta) z_{s,t}}{\theta} \frac{c_t}{s_t}, \tag{3.32}$$

which is the ratio of marginal utility of housing services to marginal utility of consumption.

Now consider again the social planning problem given by Equation (3.15) with (3.16), subject to Equations (3.17), (3.19) and the two versions of equation (3.20), replacing $y_{c,t}$ by Equation (3.18). If one denotes the respective Langrangean multipliers by $\lambda_{bc,t}$, $\lambda_{ys,t}$, $(\lambda_{bc,t} + \lambda_{kc,t})$ and $(\lambda_{bc,t} + \lambda_{ks,t})$, one can rewrite the relative price p_t as the ratio of the Lagrange multipliers for budget constraint and real estate production,

$$p_t = \frac{\lambda_{ys,t}}{\lambda_{bc,t}}. \tag{3.33}$$

To compare the model with the stylized facts presented in Section 3.2, we have to define (overall) output, house prices and stock prices. Output is defined as the sum of

[6] These definitions are given in line with the household's optimality conditions with respect to each type of capital and investment. Details are given in the stationary model derived below.

consumption good production and housing services (or real estate production),[7] both measured in units of the consumption good:

$$y_t = y_{c,t} + p_t y_{s,t}. \tag{3.34}$$

Following Hayashi (1982) and Hall (2001), the value of consumption good capital is the product of the shadow value of installed consumption good capital and the quantity of consumption good capital. In the case of no adjustment costs, the shadow value of installed consumption good capital $q_{c,t}$ is one and the value of consumption good capital equals its quantity. For nonlinear adjustment costs, $q_{c,t}$ is larger than unity and given by the ratio of the Lagrangean multiplier on capital accumulation ($\lambda_{bc,t} + \lambda_{kc,t}$) to the Langrangean multiplier on the budget constraint $\lambda_{bc,t}$,[8]

$$V_{c,t} = q_{c,t} k_{c,t}, \text{ where } q_{c,t} = \frac{\lambda_{bc,t} + \lambda_{kc,t}}{\lambda_{bc,t}}. \tag{3.35}$$

The shadow value of installed capital, also known as Tobin's q, is the value of a unit of capital in units of the consumption good. Hence, like the relative price of real estate, it can be written in terms of the Lagrange multipliers for the capital accumulation equation $\lambda_{bc,t} + \lambda_{kc,t}$ and the budget constraint. This value of consumption good capital is the total market value of the non-real estate capital stock. It is equal to all stocks and all corporate bonds outside the real estate sector. This variable will serve as the theoretical counterpart to the stock price.

We call our theoretical counterpart to real estate prices value of housing stock. It is the sum of the value of real estate capital (calculated as before) and the value of land,

$$V_{s,t} = q_{s,t} k_{s,t} + V_{L,t}, \text{ where } q_{s,t} = \frac{\lambda_{bc,t} + \lambda_{ks,t}}{\lambda_{bc,t}}. \tag{3.36}$$

Lastly, the value of land is the sum of all expected discounted future imputed land dividend payments,

$$V_{L,t} = E_0 \left[\sum_{j=0}^{\infty} \beta^t \frac{c_t}{c_{t+j}} d_{L,t+j} \right] = d_{L,t} + \beta E_t \left[\frac{c_t}{c_{t+1}} V_{L,t+1} \right], \tag{3.37}$$

where the discounting is done in terms of marginal utility of consumption.

[7] To illustrate this, one may again think of a model with real estate firms that produce and sell real estate, as mentioned above. Then, real estate production is clearly part of overall output.

[8] Notice that in our notation, $\lambda_{kc,t}$ measures the difference between the Lagrange multiplier on the budget constraint and the one on the capital accumulation equation.

3.4.2 Balanced Growth Path

To obtain a solution for the model it is convenient to reformulate it in terms of stationary variables. The appropriate transformation features the same growth rate for consumption, output of consumption goods, both types of investment and both types of capital. However, due to the fixed factor in real estate production, housing services grow more slowly.[9] This bears an important implication. As real estate is produced using one finite input factor, its balanced growth is smaller than balanced labor productivity growth. Thus, the price for real estate production and accordingly the price for housing services will rise as a result of this scarcity. Finally, land and labor feature zero growth.

Given a conjectured growth rate for each variable, we transform the model into a stationary one. Dividing all variables except for housing services s_t and the already stationary variables labor n and land L by the respective labor productivity $z_{c,t-1}$ and using a tilde to denote the detrended variables, e.g. $\tilde{c}_t \equiv c_t / z_{c,t-1}$, but $\tilde{s}_t \equiv s_t / z_{c,t-1}^{\alpha_s}$, $\tilde{p}_t \equiv p_t / z_{c,t-1}^{1-\alpha_s}$, $\tilde{k}_{i,t} \equiv k_{i,t} / z_{c,t}, i \in \{c,s\}$ and $\tilde{n}_t \equiv n_t$, $\tilde{L} \equiv L$,[10] we obtain the following representation:

$$\max_{\tilde{c}_t,\tilde{s}_t,\tilde{n}_t,\tilde{x}_{c,t},\tilde{x}_{s,t},\tilde{k}_{c,t},\tilde{k}_{s,t}} E_t \sum_{t=0}^{\infty} \beta^t \left[\theta \log \tilde{c}_t + (1-\theta) z_{s,t} \log \tilde{s}_t - A\tilde{n}_t + Z_t\right] , \tag{3.38}$$

where $Z_t = (\theta + (1-\theta) z_{s,t}) \log z_{c,t-1}$ is a stochastic scaling factor.[11] The maximization is subject to

$$\tilde{y}_{c,t} = \tilde{c}_t + \tilde{x}_{c,t} + \tilde{x}_{s,t} , \tag{3.39}$$

$$\tilde{y}_{c,t} = \tilde{k}_{c,t-1}^{\alpha_c} \tilde{n}_t^{1-\alpha_c} , \tag{3.40}$$

$$\tilde{y}_{s,t} = \tilde{s}_t = \tilde{k}_{s,t-1}^{\alpha_s} \tilde{L}^{1-\alpha_s} , \tag{3.41}$$

$$\tilde{\gamma}_t \tilde{k}_{i,t} = (1-\delta_i) \tilde{k}_{i,t-1} + g_i \left(\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}}\right) \tilde{k}_{i,t-1} , i \in \{c,s\}, \tag{3.42}$$

[9] For a comparable model with two differing growth patterns, see Greenwood et al. (1997).

[10] For the detrending procedure, it has to be taken into account that $k_{i,t-1}, i \in \{c,s\}$ is the capital stock that is valid at the beginning of period t and is thus to be divided by $z_{c,t-1}$ as well, while $k_{i,t}, i \in \{c,s\}$ is valid at the beginning of period $t+1$ and is thus to be divided by $z_{c,t}$.

[11] Note that the discount factor β is not affected, as the utility specification is logarithmic. See King et al. (1988; JME: Prod, Growth and BC I. The Basic Neoclassical Model; p. 203).

where $\tilde{\gamma}_t$ is labor productivity growth,[12]

$$\tilde{\gamma}_t = \frac{z_{c,t}}{z_{c,t-1}}.$$

The adjustment cost functions are specified so that for the detrended variables the first-order behavior of the capital accumulation is the same as in the no-adjustment-cost case:

$$g_i\left(\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}}\right) = \frac{\tilde{\delta}_i^{1/\xi_i}}{1-1/\xi_i}\left(\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}}\right)^{1-1/\xi_i} + \frac{\tilde{\delta}_i}{1-\xi_i}\,, i \in \{c, s\}.$$

The parameter $\tilde{\delta} = \delta + \bar{\gamma} - 1$ is chosen to match the investment to capital ratio in the steady state no-adjustment cost capital accumulation equation $\bar{\gamma}\bar{k}_i = (1-\delta)\bar{k}_i + \bar{x}_i$. Thus, the adjustment cost function satisfies

$$g(\tilde{\delta}) = \tilde{\delta}, \quad g'(\tilde{\delta}) = 1, \quad \xi = -\frac{g'(\tilde{\delta})}{\tilde{\delta} g''(\tilde{\delta})},$$

and $\xi > 0$ is the elasticity of the investment-capital ratio with respect to Tobin's q.[13]

The exogenous process for technology given in Equation (3.22) is transformed to

$$\log \tilde{\gamma}_t = \log \mu_t + \epsilon_{c,t}, \tag{3.43}$$

while the other exogenous processes remain the same as in Equations (3.21) and (3.23).

The household's optimal choice of consumption, housing services, labor and the two kinds of each investment and capital is obtained by setting up the usual Lagrange function. For reasons of simplification, we have replaced $y_{c,t}$ by Equation (3.40), so that we are left with four constraints. As before, we denote the Lagrange multipliers $\lambda_{bc,t}$ for the budget constraint, $\lambda_{ys,t}$ for the constraint on production of real estate, and $(\lambda_{bc,t} + \lambda_{kc,t})$, $(\lambda_{bc,t} + \lambda_{ks,t})$ for the two types of capital accumulation.[14] $\lambda_{kc,t}$ and $\lambda_{ks,t}$ measure the difference between the Lagrange multiplier on the budget constraint and the one on the respective capital accumulation equation. For linear adjustment cost functions, these multipliers are zero.

The first-order conditions for consumption, housing services, labor, the two kinds of

[12] Note that in order to have zero steady state adjustment costs, as measured by a non-binding Lagrange multiplier $\bar{\lambda}_{ki} = 0$, we have to divide all equations by the same level of productivity, i.e. $z_{c,t-1}$. Therefore, $\tilde{\gamma}_t$ has to show up on the left hand side in the capital accumulation equations.

[13] See Jermann (1998).

[14] In balanced growth, the Lagrange multiplier for the budget constraint $\lambda_{bc,t}$ and the ones for the two capital accumulation equations $(\lambda_{bc,t} + \lambda_{ki,t})$ are trending with $z_{c,t-1}^{-1}$, while the detrending of the Lagrange multiplier for real estate production is according to $\tilde{\lambda}_{ys,t} = \lambda_{ys,t}/z_{c,t-1}^{-\alpha}$.

investment and the two kinds of capital are:[15]

$$\frac{\theta}{\tilde{c}_t} = \tilde{\lambda}_{bc,t} \tag{3.44}$$

$$\frac{(1-\theta)z_{s,t}}{\tilde{s}_t} = \tilde{\lambda}_{ys,t} \tag{3.45}$$

$$A = \tilde{\lambda}_{bc,t}(1-\alpha_c)\frac{\tilde{y}_{c,t}}{\tilde{n}_t} \tag{3.46}$$

$$\tilde{\lambda}_{bc,t} = (\tilde{\lambda}_{bc,t} + \tilde{\lambda}_{kc,t})g_c'\left(\frac{\tilde{x}_{c,t}}{\tilde{k}_{c,t-1}}\right) \tag{3.47}$$

$$\tilde{\lambda}_{bc,t} = (\tilde{\lambda}_{bc,t} + \tilde{\lambda}_{ks,t})g_s'\left(\frac{\tilde{x}_{s,t}}{\tilde{k}_{s,t-1}}\right) \tag{3.48}$$

$$\begin{aligned}\tilde{\gamma}_t(\tilde{\lambda}_{bc,t} + \tilde{\lambda}_{kc,t}) = \beta E_t[\tilde{\lambda}_{bc,t+1}\alpha_c\frac{\tilde{y}_{c,t+1}}{\tilde{k}_{c,t}} + (\tilde{\lambda}_{bc,t+1} + \tilde{\lambda}_{kc,t+1})\\ \left(1-\delta_c - g_c'\left(\frac{\tilde{x}_{c,t+1}}{\tilde{k}_{c,t}}\right)\frac{\tilde{x}_{c,t+1}}{\tilde{k}_{c,t}} + g_c\left(\frac{\tilde{x}_{c,t+1}}{\tilde{k}_{c,t}}\right)\right)]\end{aligned} \tag{3.49}$$

$$\begin{aligned}\tilde{\gamma}_t(\tilde{\lambda}_{bc,t} + \tilde{\lambda}_{ks,t}) = \beta E_t[\tilde{\lambda}_{ys,t+1}\alpha_s\frac{\tilde{y}_{s,t+1}}{\tilde{k}_{s,t}} + (\tilde{\lambda}_{bc,t+1} + \tilde{\lambda}_{ks,t+1})\\ \left(1-\delta_s - g_s'\left(\frac{\tilde{x}_{s,t+1}}{\tilde{k}_{s,t}}\right)\frac{\tilde{x}_{s,t+1}}{\tilde{k}_{s,t}} + g_s\left(\frac{\tilde{x}_{s,t+1}}{\tilde{k}_{s,t}}\right)\right)]\ .\end{aligned} \tag{3.50}$$

[15]Note that the derivative of the adjustment cost function with respect to $\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}}$ equals $g_i'\left(\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}}\right) = \tilde{\delta}_i^{1/\xi_i}\left(\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}}\right)^{\frac{-1}{\xi_i}}$, $i \in \{c, s\}$.

As additional equations we get:[16]

$$\tilde{d}_{c,t} = \alpha_c \frac{\tilde{y}_{c,t}}{\tilde{k}_{c,t-1}}, \tag{3.51}$$

$$\tilde{w}_t = (1-\alpha_c)\frac{\tilde{y}_{c,t}}{\tilde{n}_t}, \tag{3.52}$$

$$\tilde{d}_{s,t} = \alpha_s \tilde{p}_t \frac{\tilde{y}_{s,t}}{\tilde{k}_{s,t-1}}, \tag{3.53}$$

$$\tilde{d}_{L,t} = (1-\alpha_s)\tilde{p}_t \frac{\tilde{y}_{s,t}}{\tilde{L}}, \tag{3.54}$$

$$\tilde{R}_{i,t+1} = \tilde{\gamma}_t^{-1} g_i'\left(\frac{\tilde{x}_{i,t}}{\tilde{k}_{i,t-1}}\right)\left(\tilde{d}_{i,t+1} + \frac{1-\delta_i + g_i\left(\frac{\tilde{x}_{i,t+1}}{\tilde{k}_{i,t}}\right)}{g_i'\left(\frac{\tilde{x}_{i,t+1}}{\tilde{k}_{i,t}}\right)} - \frac{\tilde{x}_{i,t+1}}{\tilde{k}_{i,t}}\right), i \in \{c,s\}, \tag{3.55}$$

$$\tilde{p}_t = \frac{\tilde{\lambda}_{ys,t}}{\tilde{\lambda}_{bc,t}}, \tag{3.56}$$

$$\tilde{y}_t = \tilde{y}_{c,t} + \tilde{p}_t \tilde{y}_{s,t}, \tag{3.57}$$

$$\tilde{V}_{c,t} = \tilde{\gamma}_t \tilde{q}_{c,t} \tilde{k}_{c,t}, \qquad \text{where} \quad \tilde{q}_{c,t} = \frac{\tilde{\lambda}_{bc,t} + \tilde{\lambda}_{kc,t}}{\tilde{\lambda}_{bc,t}}, \tag{3.58}$$

$$\tilde{V}_{s,t} = \tilde{\gamma}_t \tilde{q}_{s,t} \tilde{k}_{s,t} + \tilde{V}_{L,t}, \ \text{where} \quad \tilde{q}_{s,t} = \frac{\tilde{\lambda}_{bc,t} + \tilde{\lambda}_{ks,t}}{\tilde{\lambda}_{bc,t}}, \tag{3.59}$$

$$\tilde{V}_{L,t} = \tilde{d}_{L,t} + \beta E_t\left[\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\tilde{V}_{L,t+1}\right]. \tag{3.60}$$

The detrended model is given by the 20 Equations (3.39) to (3.42), (3.44) to (3.60) and the three exogenous processes (3.21), (3.23) and (3.43). After the detrending procedure, the calculation of the unique steady state of the model is straightforward. Two issues shall be stressed here. First, if $\bar{\mu} = 1$, this implies $\bar{\gamma} = 1$ and, hence, no growth in productivity. Steady state growth is given by $\bar{\mu} > 1$. Second, note that $\bar{\lambda}_{ki} = 0, i \in \{c,s\}$, i.e., in steady state the shadow price of adjustment costs is zero. This means that Tobin's q is equal to unity, and the market value equals the replacement cost of installed capital.

3.4.3 Log-linearized Model

We will approximate this system of equations around the nonstochastic steady state of the detrended variables, using the method of log-linearization. Denoting the non-

[16]Wage, land dividend, overall output and the three values have to be detrended by $z_{c,t-1}$. The usual detrending procedure does not apply to the two dividends and Tobin's q for capital, as they are a function of already detrended variables only. Returns are obtained by replacing the capital accumulation Lagrange multipliers with the first-order conditions for investment. Obviously, returns do not trend, so $\tilde{R}_{i,t+1} = R_{i,t+1}$; compare Talmain (2003) and Greenwood et al. (1997).

stochastic steady state of the detrended variable by a bar and the log deviation of a variable from its steady state by a hat, e.g. $\hat{c}_t = \log \tilde{c}_t - \log \bar{c}$, the system of equations is given in Table 3.4.3. The equations in Table 3.4.3 have the interpretation of budget constraint, production of consumption good and of real estate, capital accumulation in both sectors, first order conditions for consumption, housing services, labor, two types of investment[17] and two types of capital, i.e. the Euler equations. Then follow the equations for wage, dividends and returns, the relative price of housing services to consumption goods and overall output. The next three equations determine the values of consumption good capital, housing stock and land. Finally, the three stochastic processes for productivity, trend productivity and preferences for housing services close the system.

Notice that in log-linear terms Tobin's q is equal to $\lambda_{ki,t}$, the Lagrangean multiplier difference. Both sizes measure the shadow price of installed capital or the change in the ratio of market value of installed capital and the intrinsic asset value, see Hall (2001).

3.4.4 Results

Our baseline calibration is given in Table 3.2, in most cases it is in line with the usual values used in the real business cycle literature, see Cooley (1995). We set the steady state labor share to one third, the capital share in consumption good production to 0.36. The capital share in real estate production is set using the balanced growth properties of the model. In the model, the relative price of housing to non-housing grows more slowly than real GDP, so that the relative price is detrended $\tilde{p}_t = p_t / z_{c,t-1}^{1-\alpha_s}$. Hence, we know that relative house price growth to the power of $(1-\alpha_s)$ equals productivity and real GDP growth. Using data on average real GDP growth and growth in the CPI expenditure category housing above the growth rate of all other non-housing CPI categories, we find $\alpha_s = 0.05$ and thus the capital share in real estate production to be 95 percent. The depreciation rate for consumption good capital is set to 2.5 percent per quarter, that for real estate capital to half of that, as real estate is typically more durable. The share of consumption in the utility function determines also the share of expenditures on consumption relative to housing services. In U.S. CPI data housing has a relative importance of 42 percent of all expenditures, so we set $\theta = 0.58$. For the capital adjustment cost parameter, we follow Jermann (1998) and set $\xi_c = 0.23$ for the consumption good sector, whereas for real estate we assume higher adjustment cost

[17] For these log-linear approximations, note that $\hat{\lambda}_{kc}$ and $\hat{\lambda}_{ks}$ cannot be defined in the usual way, as their steady state value equals zero. Instead, we define e.g. $\hat{\lambda}_{ks} = \frac{\lambda_{ks} - \bar{\lambda}_{ks}}{\bar{\lambda}_{bc} + \bar{\lambda}_{ks}} = \frac{\lambda_{ks}}{\bar{\lambda}_{bc}}$.

Table 3.1: The log-linearized Model

$$\hat{y}_{c,t} = \frac{\bar{c}}{\bar{y}_c}\hat{c}_t + \frac{\bar{x}_c}{\bar{y}_c}\hat{x}_{c,t} + \frac{\bar{x}_s}{\bar{y}_c}\hat{x}_{s,t} \tag{3.61}$$

$$\hat{y}_{c,t} = \alpha_c \hat{k}_{c,t-1} + (1-\alpha_c)\hat{n}_t \tag{3.62}$$

$$\hat{y}_{s,t} = \hat{s}_t = \alpha_s \hat{k}_{s,t-1} \tag{3.63}$$

$$\bar{\gamma}\hat{k}_{c,t} = (1-\delta_c)\hat{k}_{c,t-1} + \tilde{\delta}_c \hat{x}_{c,t} - \bar{\gamma}\hat{\gamma}_t \tag{3.64}$$

$$\bar{\gamma}\hat{k}_{s,t} = (1-\delta_s)\hat{k}_{s,t-1} + \tilde{\delta}_s \hat{x}_{s,t} - \bar{\gamma}\hat{\gamma}_t \tag{3.65}$$

$$\hat{c}_t = -\hat{\lambda}_{bc,t} \tag{3.66}$$

$$\hat{s}_t = -\hat{\lambda}_{ys,t} + \hat{z}_{s,t} \tag{3.67}$$

$$\hat{n}_t = \hat{\lambda}_{bc,t} + \hat{y}_{c,t} \tag{3.68}$$

$$\hat{\lambda}_{kc,t} = \hat{q}_{c,t} = \frac{1}{\xi_c}(\hat{x}_{c,t} - \hat{k}_{c,t-1}) \tag{3.69}$$

$$\hat{\lambda}_{ks,t} = \hat{q}_{s,t} = \frac{1}{\xi_s}(\hat{x}_{s,t} - \hat{k}_{s,t-1}) \tag{3.70}$$

$$0 = E_t[\hat{\lambda}_{bc,t+1} - \hat{\lambda}_{bc,t} - \hat{\gamma}_t + \hat{R}_{c,t+1}] \tag{3.71}$$

$$0 = E_t[\hat{\lambda}_{bc,t+1} - \hat{\lambda}_{bc,t} - \hat{\gamma}_t + \hat{R}_{s,t+1}] \tag{3.72}$$

$$\hat{d}_{c,t} = \hat{y}_{c,t} - \hat{k}_{c,t-1} \tag{3.73}$$

$$\hat{w}_t = \hat{y}_{c,t} - \hat{n}_t \tag{3.74}$$

$$\hat{d}_{s,t} = \hat{p}_t + \hat{y}_{s,t} - \hat{k}_{s,t-1} = (\alpha_s - 1)\hat{k}_{s,t-1} \tag{3.75}$$

$$\hat{d}_{L,t} = \hat{p}_t + \hat{y}_{s,t} \tag{3.76}$$

$$\hat{R}_{c,t} = \frac{\bar{R}_c - 1 + \delta_c}{\bar{R}_c}\hat{d}_{c,t} - \hat{\lambda}_{kc,t-1} + \frac{\bar{\gamma}}{\bar{R}_c}\hat{\lambda}_{kc,t} \tag{3.77}$$

$$\hat{R}_{s,t} = \frac{\bar{R}_s - 1 + \delta_s}{\bar{R}_s}\hat{d}_{s,t}) - \hat{\lambda}_{ks,t-1} + \frac{\bar{\gamma}}{\bar{R}_s}\hat{\lambda}_{ks,t} \tag{3.78}$$

$$\hat{p}_t = \hat{\lambda}_{ys,t} - \hat{\lambda}_{bc,t} \tag{3.79}$$

$$\hat{y}_t = \frac{\bar{y}_c}{\bar{y}}\hat{y}_{c,t} + \frac{\bar{p}\bar{y}_s}{\bar{y}}\hat{p}_t + \frac{\bar{p}\bar{y}_s}{\bar{y}}\hat{s}_t \tag{3.80}$$

$$\hat{V}_{c,t} = \hat{q}_{c,t} + \hat{k}_{c,t} + \hat{\gamma}_t \tag{3.81}$$

$$\frac{\bar{V}_s}{\bar{V}_s - \bar{V}_L}\hat{V}_{s,t} = \hat{q}_{s,t} + \hat{k}_{s,t} + \hat{\gamma}_t + \frac{\bar{V}_L}{\bar{V}_s - \bar{V}_L}\hat{V}_{L,t} \tag{3.82}$$

$$0 = E_t[-\hat{V}_{L,t} + \beta\hat{V}_{L,t+1} - \beta\hat{c}_{t+1} + \beta\hat{c}_t + (1-\beta)\hat{p}_t + (1-\beta)\hat{s}_t] \tag{3.83}$$

$$\hat{\gamma}_t = \hat{\mu}_t + \epsilon_{c,t}, \tag{3.84}$$

$$\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \nu_t, \tag{3.85}$$

$$\hat{z}_{s,t} = \rho_s \hat{z}_{s,t-1} + \epsilon_{s,t}. \tag{3.86}$$

Table 3.2: Baseline Calibration

Parameter	Value	Explanation
$\bar{N}$	0.333	Steady state employment is $\frac{1}{3}$ of total time endowment
α_c	0.36	Capital share in consumption good production
α_s	0.95	Capital share in real estate production
δ_c	0.025	Depreciation rate for capital
δ_s	0.012	Depreciation rate for real estate capital
β	1/1.01	Discount factor
θ	0.58	Share of consumption in the household's utility
ξ_c	0.23	Capital adjustment cost curvature parameter (0=capital fixed, ∞=no cost)
ξ_s	0.12	Real estate capital adjustment cost curvature parameter
ρ_s	0.95	Autocorrelation of preference shock
$\bar{\mu}$	1.006	Steady state technology trend
ρ_μ	0.99	Autocorrelation of trend technology
σ_{ϵ_c}	0.712	Standard deviation of technology shock in percent
σ_{ϵ_s}	0.407	Standard deviation of housing preference shock in percent
σ_{ϵ_v}	0.712	Standard deviation of trend technology shock in percent

curvature of $\xi_s = 0.12$. The preference shock is assumed to have an AR(1) coefficient of .95 and a standard deviation of 0.407, the unit root technology process has standard deviation of 0.712 percent, like the trend technology process, which has an AR(1) coefficient of 0.99. The mean of the trend technology growth is set to 1.006, which implies a steady state real quarter to quarter growth of 0.6 percent.

The model is solved using standard algorithms, see Uhlig (1999). The results of the model are presented in two ways: First, we show the behavior of the detrended variables. Second, we re-transform the variables to again include the specific trend that has been removed earlier, as proposed in Uhlig (2003).

For the discussion of results we focus on impulse responses of GDP, relative prices and the values of land, housing and consumption good capital. The impulse responses of these variables in detrended form to preference and the various productivity shocks are given in Figures 3.6 to 3.9. A shock in housing preferences makes housing services more valuable. Thus, both, the relative price of housing services p_t and Tobin's q for housing rise on impact. Accordingly, the values of land and of housing rise. As future real estate production can be financed only by means of consumption good production, Tobin's q for consumption good capital and overall output rise as well, but to a lesser extent. So both, the ratios of housing value to GDP and of housing value to the value of the

Figure 3.6: Impulse response to a shock in housing preferences

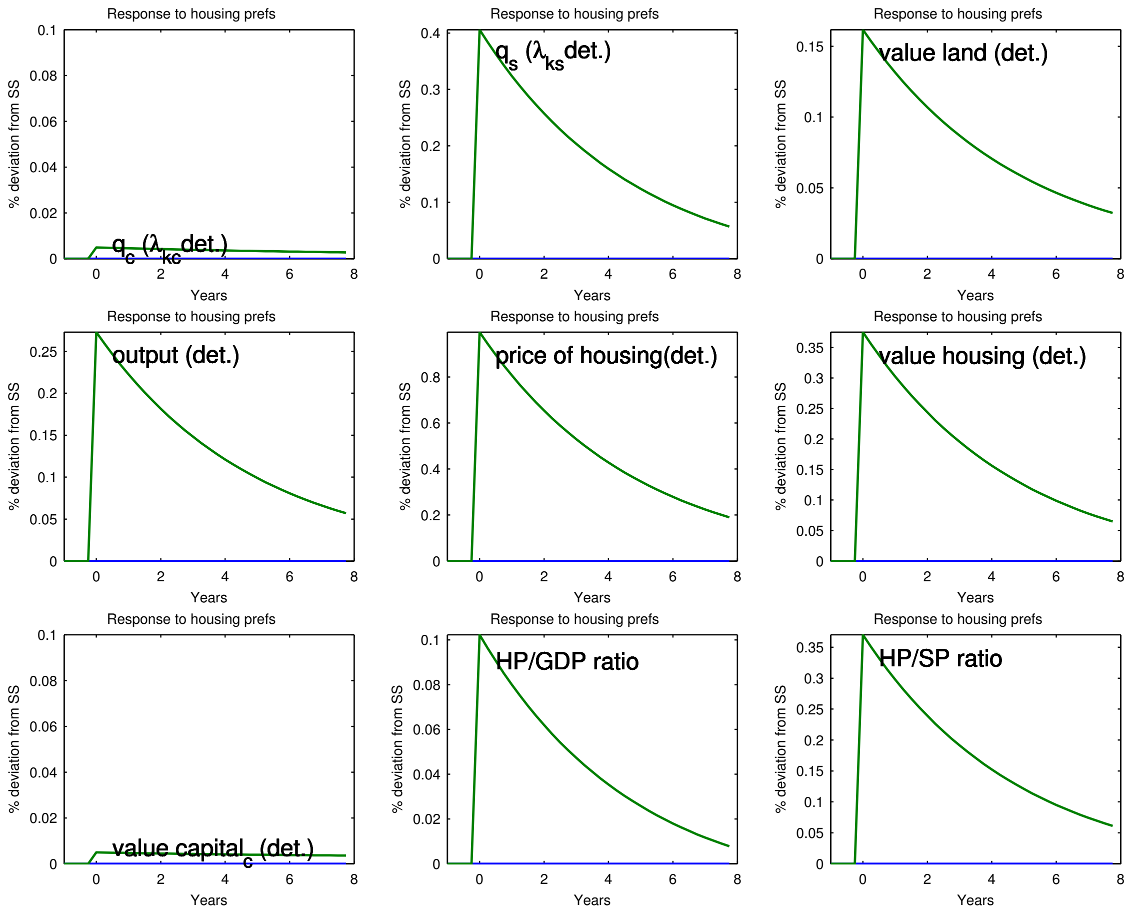


Figure 3.7: Impulse response to a shock in the level of technology

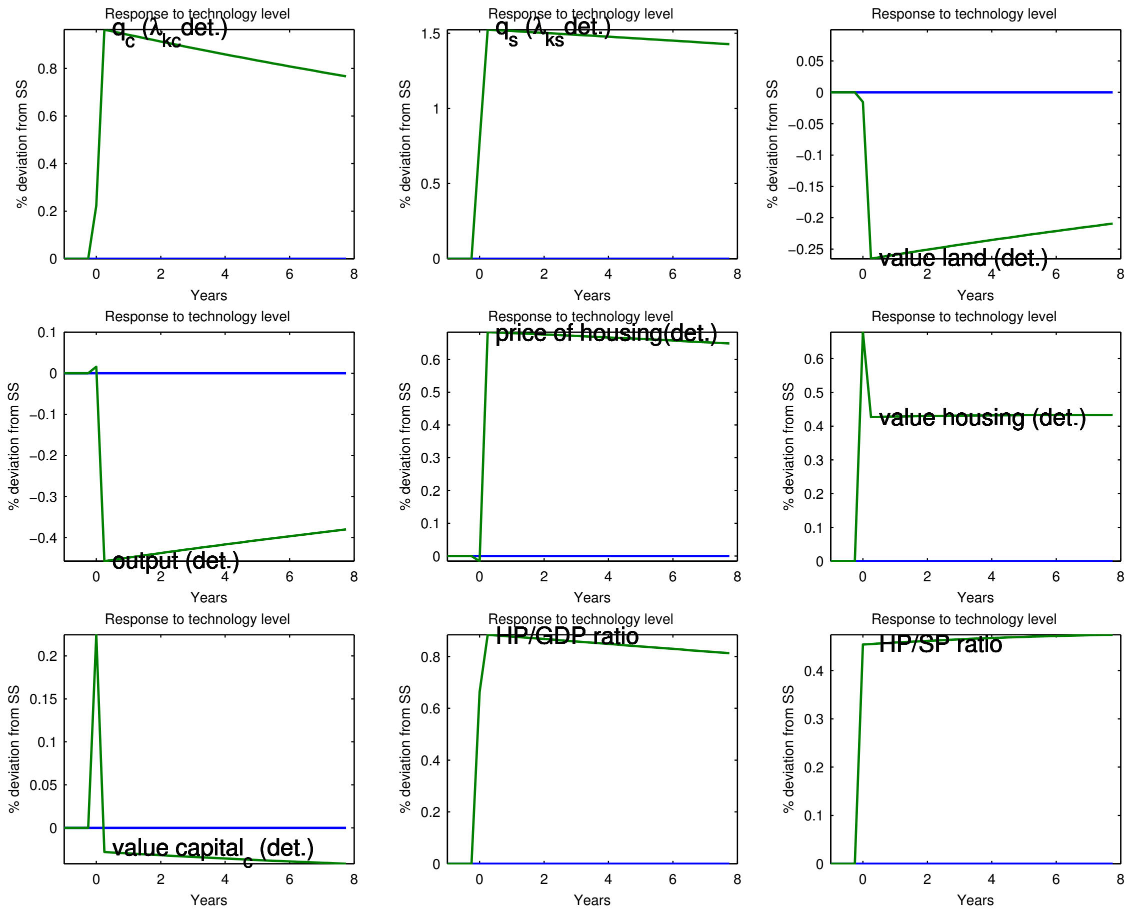


Figure 3.8: Impulse response to a shock in technology growth

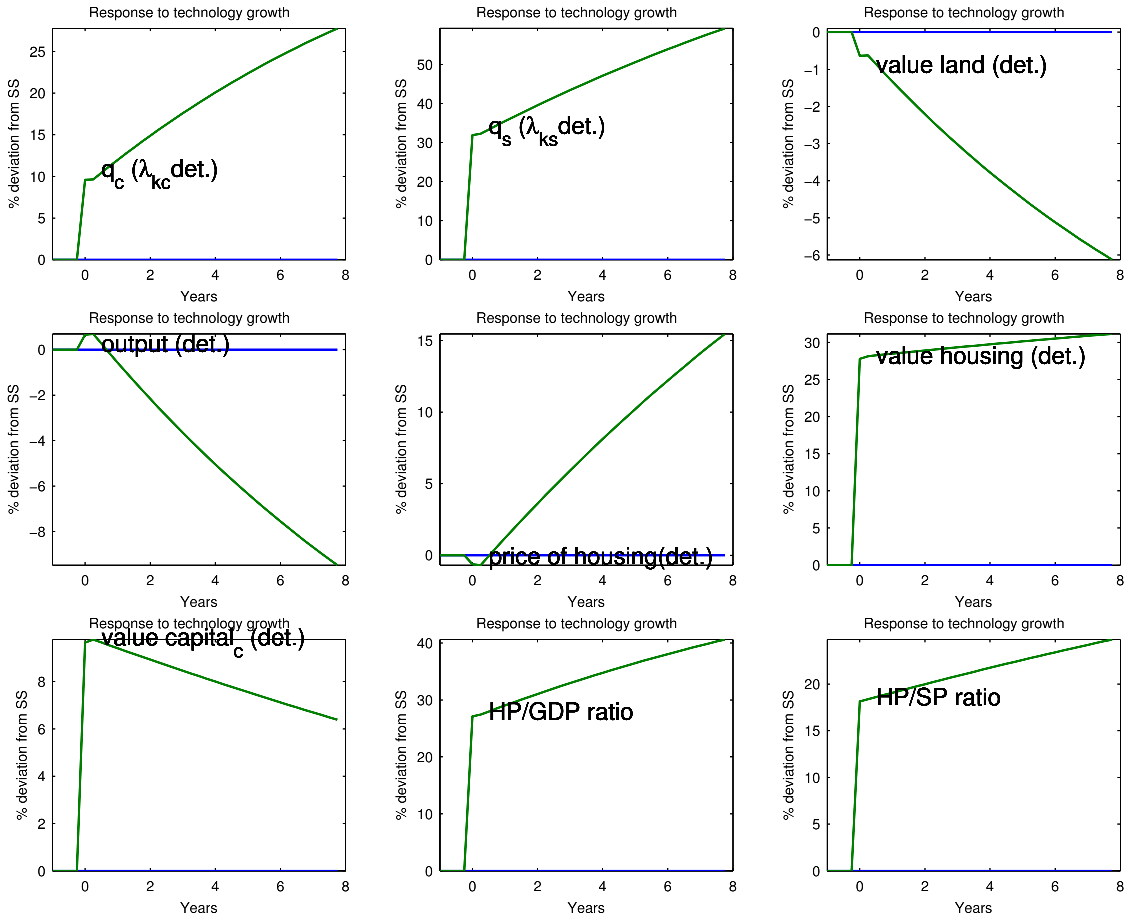
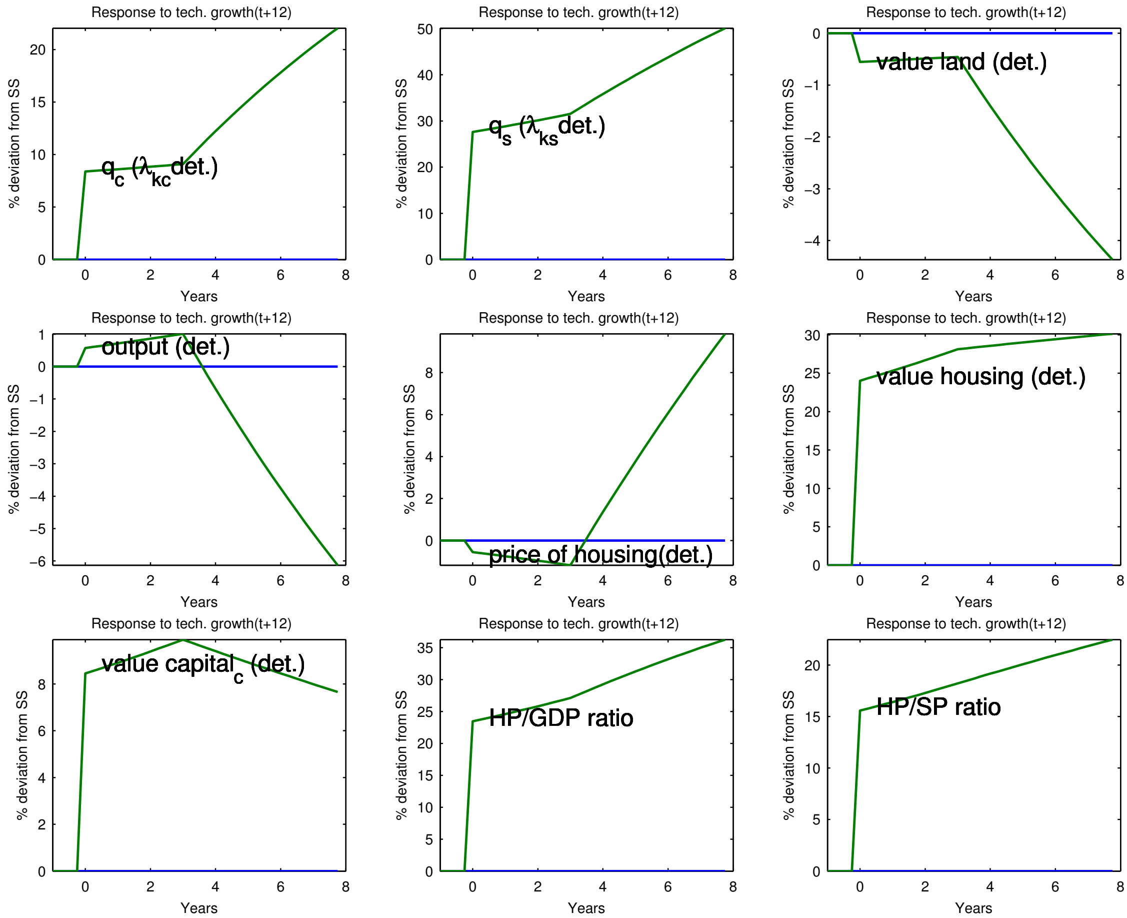


Figure 3.9: Impulse response to a shock in technology growth twelve periods ahead



capital stock in the consumption good sector rise on impact and then fall to return to their steady state.

A shock to the level of technology leads to highly persistent responses. The market values of both types of capital and, hence, both types of Tobin's q increase significantly. But as capital adjustment is costly and thus takes place only gradually, the value of non-housing capital and output grow only slowly, slower than the trend. Consequently, the detrended variables fall on impact. But as the adjustment costs in the real estate sector are assumed to be bigger, Tobin's q is higher in that sector, and the relative price as well as the value of housing are above their steady state for a long time. For this reason, the two ratios are again both positive, just like after the preference shock.

A shock to technology growth, i. e., a positive v_t, implies a higher trend growth rate for a long time. Hence, the positive effect on Tobin'S q is big, increasing and prolonged compared to the level shock. After an initial period of higher trend growth, output grows more slowly than the trend, i.e., it falls below steady state, and the value of land is continuously growing more slowly than the trend. In contrast to this, the values of housing and of consumption good capital are above their steady state. Note in particular that the steady-state deviation of the value of housing increases more than 25 percent. Therefore, the house price to GDP ratio and the house price to stock price ratio on impact increase by more than 25 and more than 15 percent, respectively. These numbers are close to what we have seen in the data in Section 3.2.

If a future shock to technology growth is anticipated today, the responses on impact are nearly as big as if the shock hit today. The responses then increase until the shock hits. As an example, Figure 3.15 depicts a technology growth shock anticipated three years in advance. Here, the increases in the house price to GDP and the house price to stock price ratios are between 25 and 15 percent.

Comparing the effect of a current technology growth shock to an anticipated technology growth shock in three years on the house price to GDP and the house price to stock price ratios, one is struck by the similarities. Looking at Figures 3.10 and 3.11, one can see that the anticipation of a future shock has on impact nearly as big effects as the actual shock. Of course, this result heavily depends on the assumed adjustment cost function. The higher the curvature of this function, the bigger the intertemporal smoothing effect and, hence, the larger the immediate increase in the the valuation of real estate.

The impulse responses of the different types of output, the two capital stocks, the relative price of housing and the three values with the trend again added are given in Fig-

Figure 3.10: Impulse response to a shock in technology growth

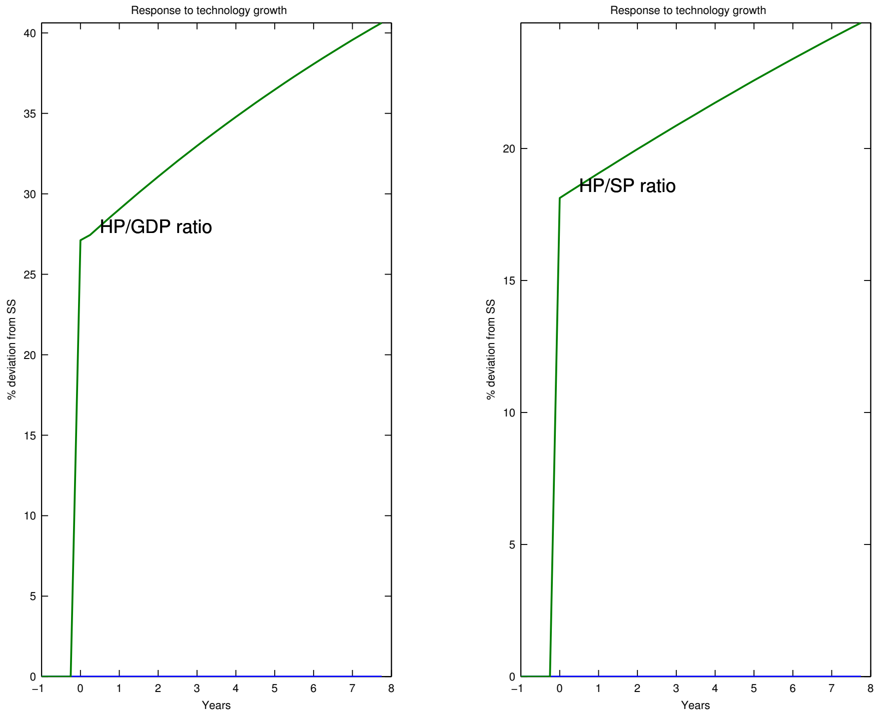


Figure 3.11: Impulse response to a shock in technology growth twelve periods ahead

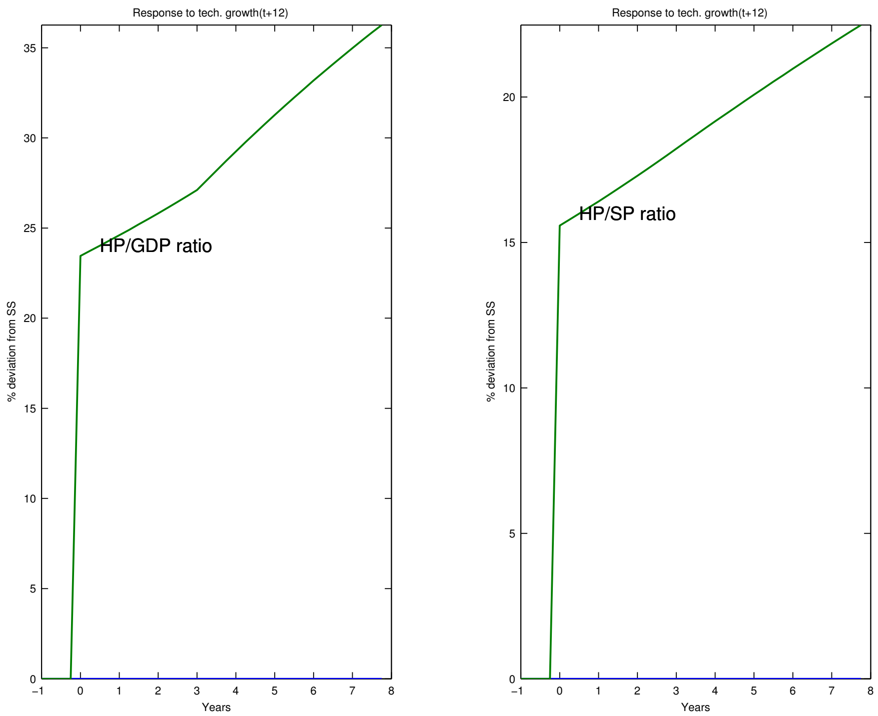


Figure 3.12: Impulse response to a shock in housing preferences, including trend

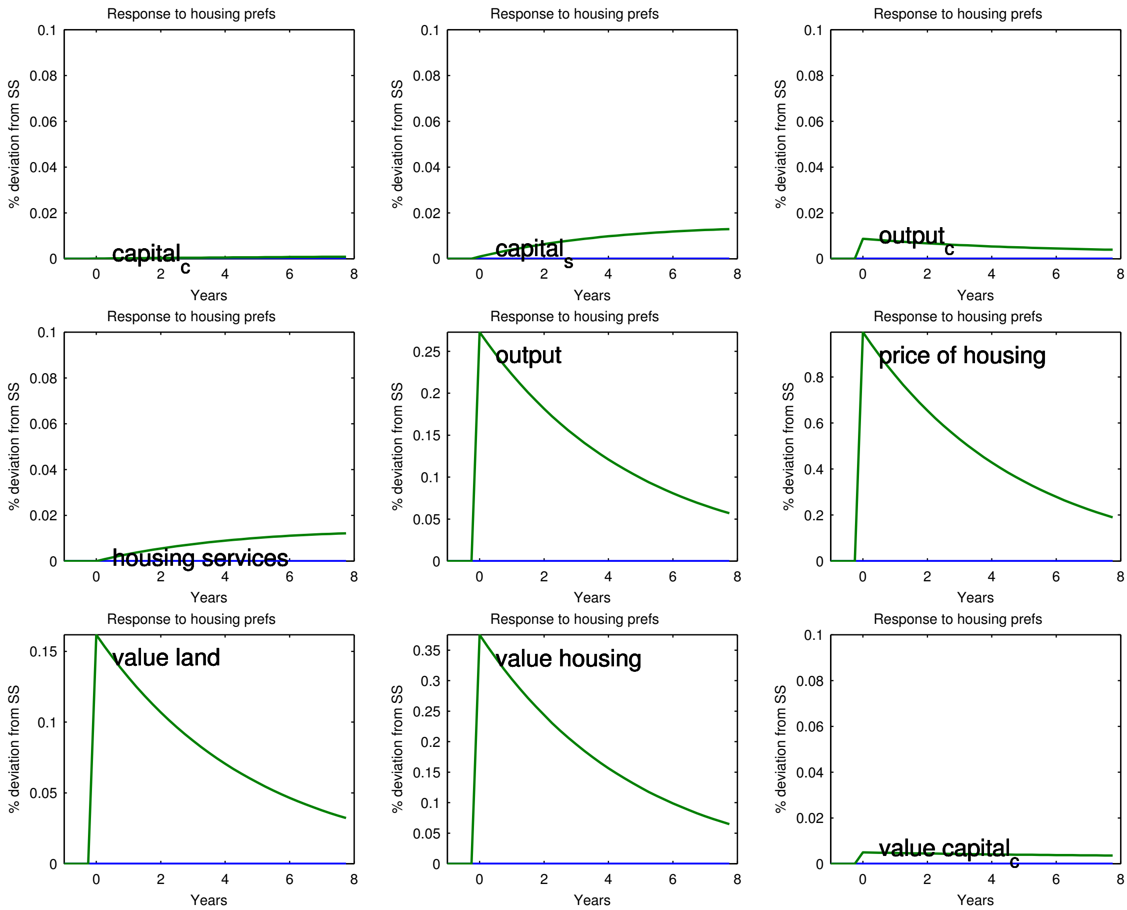


Figure 3.13: Impulse response to a shock in the level of technology, including trend

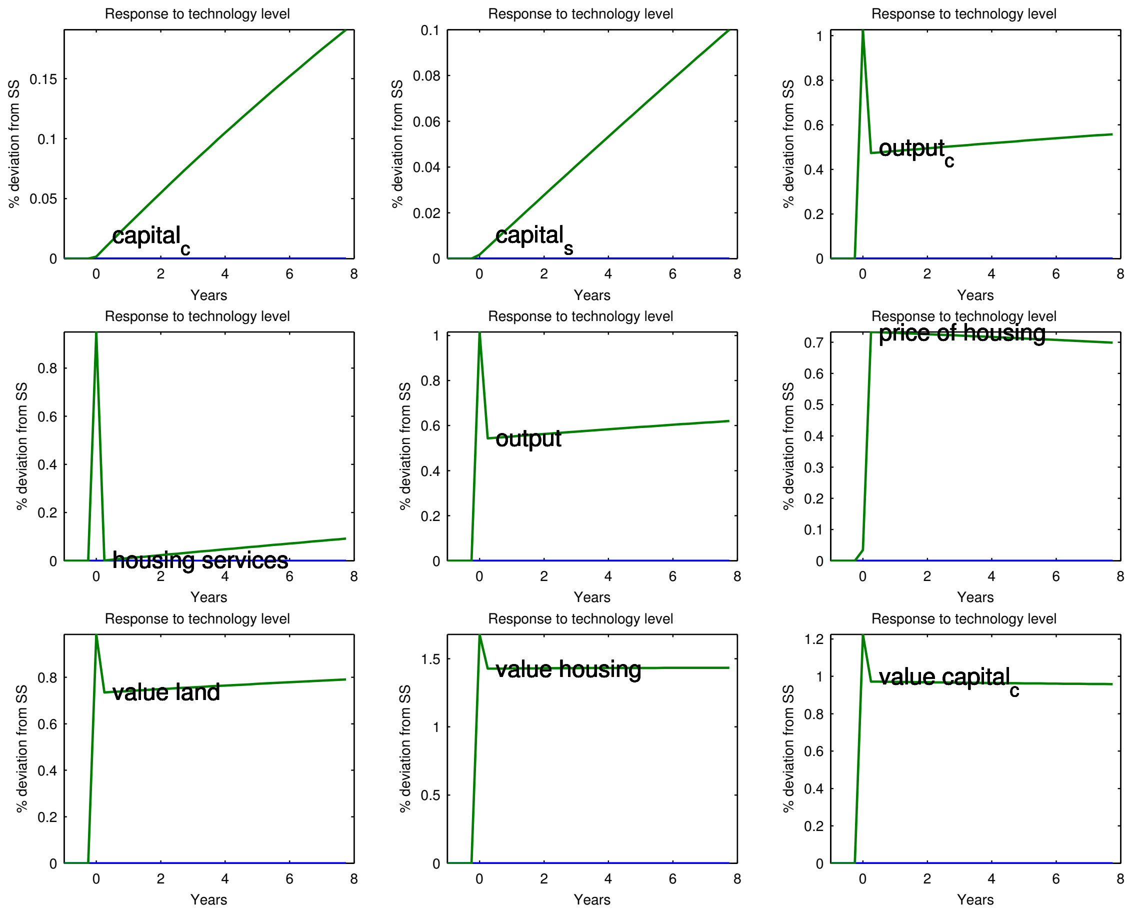


Figure 3.14: Impulse response to a shock in technology growth, including trend

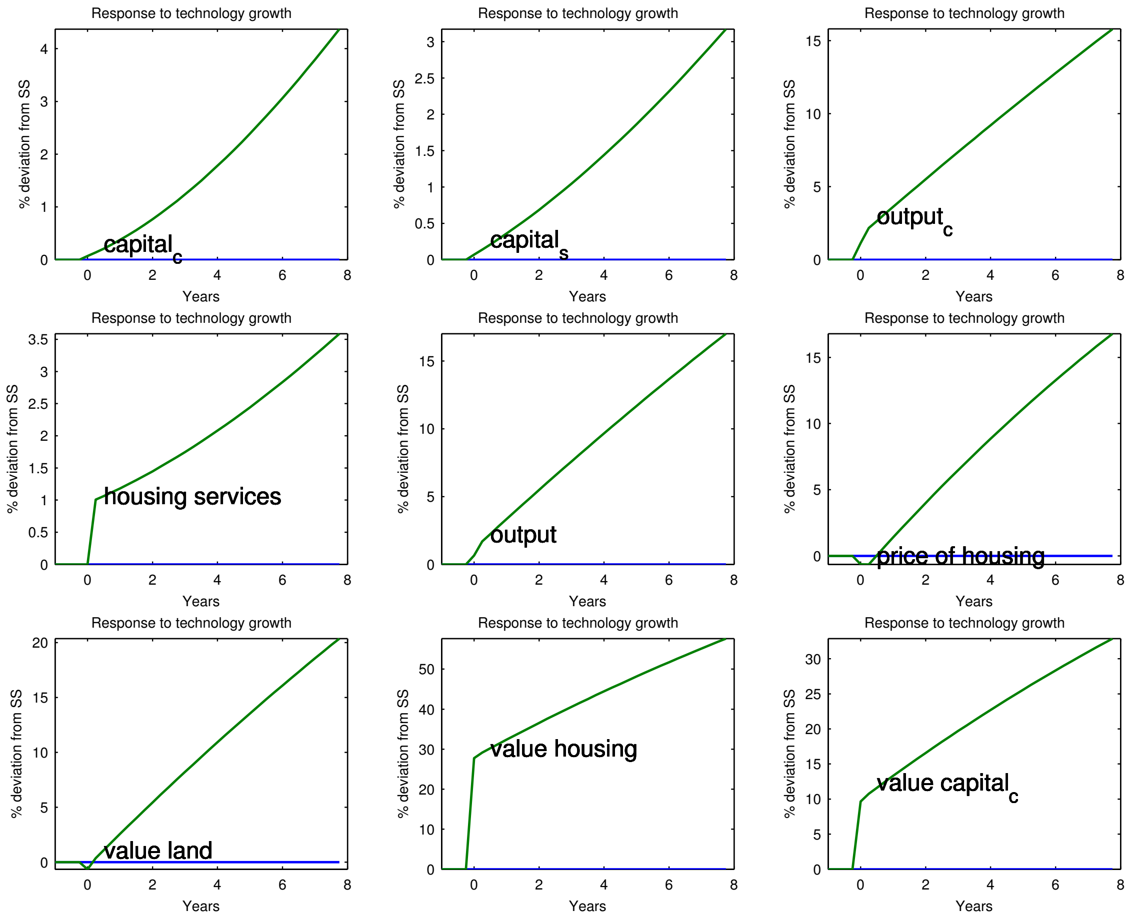


Figure 3.15: Impulse response to a shock in technology growth twelve periods ahead, including trend

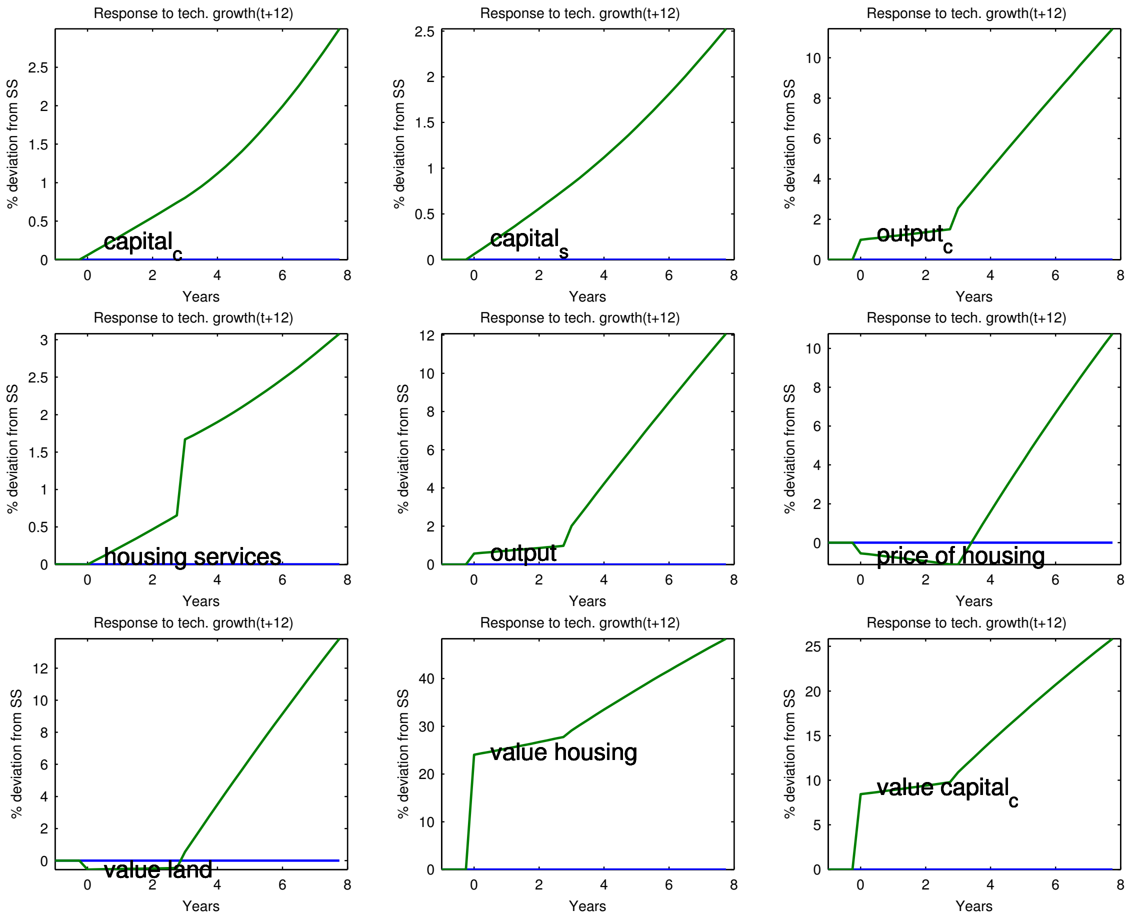
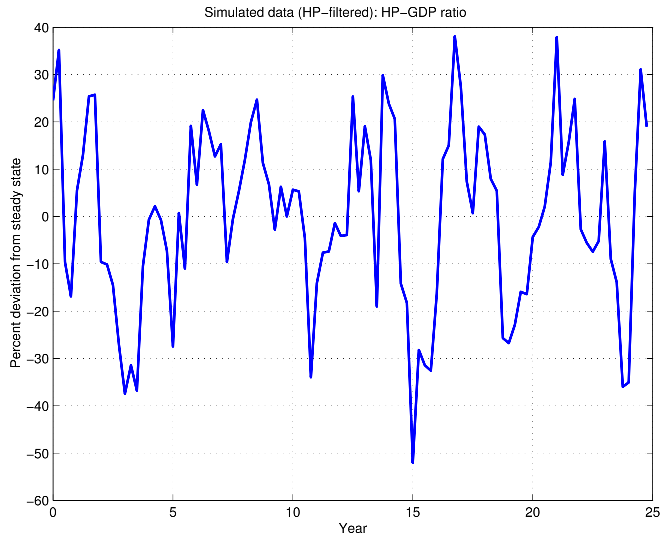


Figure 3.16: Simulated data for the house price to GDP ratio, HP-filtered, averages over 50 simulations



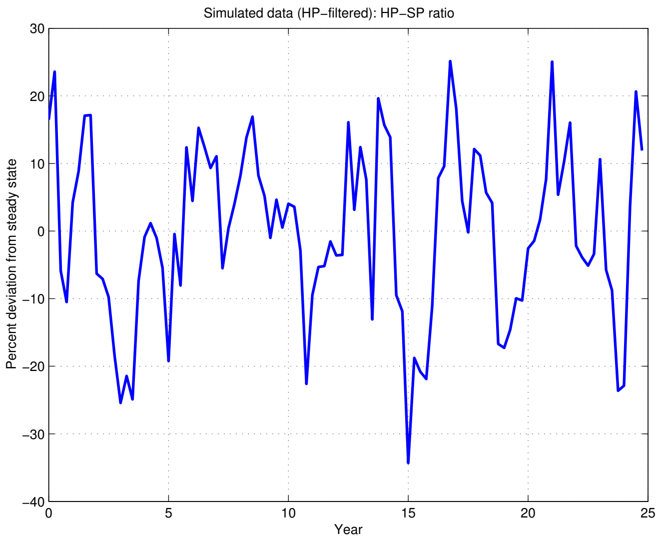
ures 3.12 to 3.15. The figures show the generally positive effect of all shocks on these trending variables.

As we have seen, in this model future productivity growth leads to an immediate boom in the value of housing and the value of the capital stock in the consumption good sector. Using stock prices and house prices as proxies for these values, we find a stock price boom and a house price boom today in response to an expected future technology growth shock. Put it the other way round, a situation in which both house price to GDP ratio and house price to stock price ratio move upward could be the result of an increase in the expected future technology growth. In other words, positive economic forecasts may effect house prices more than proportionately.

How do simulated time series of the house price to GDP and the house price to stock price ratios look like? are they comparable to what we have seen in the data? Figures 3.16 and 3.17 provide a first answer. We see swings of enormous size, though their amplitude is still smaller than in actual data, and so is the persistence of the series. Regarding the correlation pattern, the data provided in Section 3.2 features contemporaneous correlation coefficients of the house price to GDP ratio with the house price to stock price ratio between 0.17 for the U.S. and 0.87 for the U.K., with Germany (0.48) and Japan (0.30) in between. The HP-filtered model equivalent coefficient is 0.95.[18]

[18]The results provided in the last paragraph should be seen as tentative in so far as a direct comparison between log indexed data and HP-filtered model simulations might well be misleading.

Figure 3.17: Simulated data for the house price to stock price ratio, HP-filtered, averages over 50 simulations



3.5 Conclusion

A simple two sector RBC model can be used to address issues in real estate economics. Assuming real estate production to be restricted by the finite factor land, whereas the production of other goods is not restricted in this way, the relative price of real estate has to increase over time as the economy grows. In particular, we have shown that in the presence of capital adjustment costs, a shock to present or expected future productivity growth results in a sudden and sharp increase in the value of real estate. The result of this kind of shock is a simultaneous and big increase in the house price to GDP ratio and in the house price to stock price ratio. The implied positive correlation between the two ratios is generally confirmed by the data for aa four investigated countries: U. S., U. K., Japan and Germany.

One might argue that the high house price to GDP ratio in the U. S. and the U. K. until recently, as documented in the middle panel of Figure 3.5 is the result of high expected future productivity growth rates in these countries, whereas the low ratios in Japan and Germany come from less optimistic expectations. Seen in this light, the current downturn in house prices in the U. S. and the U. K. is the effect of anticipated positive shocks that did not materialize or that are no longer expected to materialize in the future. We regard this line of research to be promising.

4 Putting up a Good Fight: The Galí-Monacelli Model versus "The Six Major Puzzles in International Macroeconomics"

*In this paper, the following question is posed: Can the New Keynesian Open Economy Model by Galí and Monacelli (2005b) explain "Six Major Puzzles in International Macroeconomics", as documented in Obstfeld and Rogoff (2000*b*)?*

The model features a small open economy with complete markets, Calvo sticky prices and monopolistic competition. As extensions, I explore the effects of an estimated Taylor rule and additional trade costs. After translating the six puzzles into moment conditions for the model, I estimate the five most effective parameters using simulated method of moments (SMM) to fit the moment conditions implied by the data. Given the simplicity of the model, its fit is surprisingly good: among other things, the home bias puzzles can easily be replicated, the exchange rate volatility is formidably increased and the exchange rate correlation pattern is relatively close to realistic values. Trade costs are one important ingredient for this finding.

4.1 Introduction

Nowadays, the New Keynesian dynamic stochastic general equilibrium (DSGE) paradigm is the basis for most open economy macroeconomic models.[1] Since Obstfeld and Rogoff (1995), models with a small set of shocks and frictions are widely used for the

[1] Instead of New Keynesian, the labels New Neoclassical Synthesis and – especially for the open economy – New Open Economy Macroeconomics are used interchangeably. A survey on New Open Economy Models can be found in Lane (2001).

analysis of policies, especially monetary policy. The comparative simplicity of these models has two implications. On the one hand, the working mechanisms of these models are easily understood. On the other hand, the connection between these stylized models and real world problems can be easily questioned. Researchers have reacted to this in two ways. First, they have built New Keynesian DSGE models with more shocks and frictions. Adolfson, Laséen, Lindé, and Villani (2005) and the IMF's Global Economy Model, as presented in Pesenti (2008) are good examples for this approach, and more are to come. Loosing some of their simplicity and tractability, these papers gain in terms of realism and applicability. Second, researchers have tried to assess the actual quality of the stylized models when confronted with the data, or at least with specific aspects of it. Chari, Kehoe, and McGrattan (2002) and Lubik and Schorfheide (2007) are two examples for this approach.

In this paper, this second way is followed. A specific stylized New Keynesian DSGE model is confronted to a specific set of first and second moments of international macroeconomic data. The model used is the one by Galí and Monacelli (2005b). This model is also reprinted in the textbook by Galí (2008) and can be regarded as a prototype of New Keynesian Open Economy Models.[2] The main components of this kind of models are a forward looking Phillips curve, a dynamic IS-curve and Calvo (1983) sticky prices. The open economy assumptions in this model are a small open economy versus the rest of the world, modeled as the limiting case of a two country world with one country infinitely small such that it does not influence the other, producer currency pricing, and complete financial markets. I modify the model in three respects. First, I disregard the multi-country framework, as is done in previous versions of that paper, see Galí and Monacelli (2002), henceforth GM. Second, besides the four monetary policy rules analyzed in Galí and Monacelli (2005b), I include an alternative Taylor rule monetary policy as in Clarida et al. (1998) which is more suitable for estimation issues. Third, I allow for the possibility of costs to trade in goods, following the suggestion by Obstfeld and Rogoff (2000*b*).[3]

Regarding the data, I focus on the "Six Major Puzzles in International Macroeconomics" as presented in Obstfeld and Rogoff (2000*b*), henceforth OR. These are (1) the home bias in trade puzzle, (2) the high investment-savings correlation, (3) the home bias in equity portfolio puzzle, (4) the low international consumption correlation, (5) the purchasing power parity puzzle and (6) the exchange rate disconnect puzzle. In ap-

[2] McCallum and Nelson (2001, p. 10) call this model a "standard" model that they use as a benchmark with which to compare their own model.

[3] Thus, I am putting Obstfeld and Rogoff's idea to a test "in a much richer framework featuring imperfect competition plus sticky prices". See Obstfeld and Rogoff (2000*b*, pp. 340f.).

plying the GM model – extended for trade costs – to the OR puzzles this paper features a second motivation: while Obstfeld and Rogoff only sketch their idea of the effects of trade costs, this paper features a complete DSGE analysis of these effects.

For different sets of parameters, three different procedures are applied in order to “take the model to the data": First, I calibrate those parameters that have agreed-upon values or that are unimportant with respect to the six puzzles. In a next step, I estimate the Taylor rule parameters using generalized method of moments (GMM). Also, I use estimates for the assumed exogenous processes. In this step, I follow Galí and Monacelli (2005b, p. 723) in using data for Canada as “a prototype small open economy". The third and last procedure is simulated method of moments (SMM). This method is used to set the five most important parameters such that the distance between model moments and the data moments from the six puzzles is minimized. The parameters are those for trade costs, degree of openness, Calvo price stickiness, the international elasticity of substitution and relative risk aversion.

I come to the conclusion that the model can easily explain puzzles (1) and (3), thanks to the combination of trade costs and the degree of openness parameter, the “home bias in preferences" parameter mentioned in OR. The investment-savings puzzle is addressed only indirectly by means of a relation between net exports and the real interest rate, where the expected negative correlation is reproduced. The biggest deficiency of the model is that international output correlation is way too low, and the real exchange rate volatility and its correlation pattern is not exactly in line with the data.

Compared to a case without trade costs and without degree of openness parameter, the combination of the two elements leads to better results for all the puzzles. Very high values for the two home bias puzzles (1) and (3) can be replicated. The result of puzzle (2) remains stable, but it is now possible to also address the last three puzzles to some degree. The high exchange rate volatility of the data can be achieved by a combination of a high risk aversion as in Chari, Kehoe, and McGrattan (2002), sizeable trade costs and a low degree of openness. The “disconnectedness" of real exchange rate volatility, i.e., the fact that real exchange rates are by far more volatile than any other macroeconomic aggregate – one part of the “disconnect" puzzle – is reproduced relatively well. But the second dimension of the “disconnect" puzzle, i.e., the low correlation between the real exchange rate and all other macroeconomic aggregates, is not explained by the model. Instead, the model features a positive correlation between the real exchange rate and output. The biggest weakness of the model is with respect to the international consumption correlations relative to the international output correlation. Output is by far not enough correlated internationally in the model.

The paper continues as follows. In Section 2, the model is presented. Section 3 briefly sketches the puzzles and the implied moments for the parametrization process. Section 4 explains the parametrization methods and choices. Results are presented in Section 5. Section 6 concludes the paper.

4.2 Model

4.2.1 Environment

There are two countries, the home country (H) and the foreign country (the "rest of the world", F). If not indicated differently, the following applies to both of them, whereas foreign variables are denoted by an asterisk. There are infinitely long living households, which experience utility from consumption of home and of foreign goods. Firms produce in monopolistic competition, and governments collect taxes, pay transfers and conduct monetary policy with an interest rate rule. The same applies to the foreign economy, except for the fact that foreign households' consumption of home goods is negligibly small for them. While international financial markets are complete, there is a friction in the goods market: Transportation of goods from one country to another decreases its quantity by the factor κ, which can be understood as "iceberg melting".

Preferences

A representative household decides about its expected infinite labor supply and consumption to maximize its utility, which is assumed to be separable between the two elements consumption C_t and hours of labor N_t:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - V(N_t) \right] , \tag{4.1}$$

where U is defined as $U(C_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma}$ and V as $V(N_t) \equiv \frac{N_t^{1+\varphi}}{1+\varphi}$. The parameters used are discount factor β, constant of relative risk aversion σ and elasticity of labor supply $1/\varphi$. Consumption C_t is composed of

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} . \tag{4.2}$$

$C_{F,t}$ and $C_{H,t}$ are indices related to the consumption of foreign and domestic products, respectively, which are themselves integrals over all firms $i \in [0;1]$:

$$C_{j,t} = \left(\int_0^1 C_{j,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad , \; j \in \{H;F\}, \tag{4.3}$$

with η being the elasticity of substitution between domestic and foreign goods, and ε the elasticity of substitution between goods of the same country.

Endowment

Each household is endowed with one unit of time per period.

Technology

Each domestic firm $i \in [0;1]$ produces its output $Y_t(i)$ with production technology $Y_t(i) = A_t N_t(i)$, where $\log(A_t) = a_t = \rho_a a_{t-1} + \epsilon_t$ is stochastic productivity. To simplify matters, production in the rest of the world is assumed to evolve exogenously according to $\log(Y_t^*) - \log \bar{Y}^* = y_t^* = \rho_y^* y_{t-1}^* + \epsilon_t^*$.

Information

Households have complete information until and including the current period, and they have rational expectations about future periods. The same applies to firms and governments.

4.2.2 Competitive Equilibrium

Households work at firms in their own country, pay lump-sum taxes, and trade nominal bonds which include shares in firms of all countries. They have access to a complete set of internationally traded contingent claims. Firms hire labor, produce, and sell their goods at home and abroad under monopolistic competition. They set prices for all markets in domestic currency (producer currency pricing) according to the Calvo (1983) price stickiness. Finally, they receive a wage subsidy τ. Governments receive lump- sum taxes T_t, pay wage subsidies, and are not allowed to accumulate debt. Monetary policy is made by setting the nominal interest rate.

Competitive Equilibrium: Households

The budget constraint domestic households are faced with each period t is

$$\int_0^1 [P_{H,t}(i)C_{H,t}(i) + P_{F,t}(i)C_{F,t}(i)]di + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_t N_t + T_t \,, \tag{4.4}$$

with $Q_{t,t+1}$ the stochastic discount factor for nominal payoffs, related to the gross return R_t by $E_t(Q_{t,t+1}) = \frac{1}{R_t}$. D_{t+1} is the nominal payoff in period $t+1$ of a portfolio held at the end of period t. This portfolio includes shares in firms, and its payoff is *cum dividend*. As markets are complete, there is a complete set of state-contingent claims, traded internationally. W_t is the nominal wage and T_t a lump-sum transfer or tax. Foreign households similarly face

$$\int_0^1 [P^*_{H,t}(i)C^*_{H,t}(i) + P^*_{F,t}(i)C^*_{F,t}(i)]di + E_t\left[\frac{Q_{t,t+1}D_{t+1}}{\mathscr{E}_{t+1}}\right] \leq \frac{D_t}{\mathscr{E}_t} + W^*_t N^*_t + T^*_t \,, \tag{4.5}$$

with an asterisk denoting a foreign variable and $\mathscr{E}_t$ the nominal exchange rate, defined as the price of foreign currency in terms of home currency.

Price indices are the result of expenditure minimization for a given level of consumption. This minimization leads to the following outcomes: The consumer price index (CPI) comprises all consumption goods, i.e., domestic and foreign goods, and is given by

$$P_t \equiv [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}} \,. \tag{4.6}$$

$P_{H,t}$ and $P_{F,t}$ are the price indices of domestic and foreign goods, respectively, and are given by

$$P_{j,t} \equiv \left(\int_0^1 P_{j,t}(i)^{1-\varepsilon}di\right)^{\frac{1}{1-\varepsilon}} \quad \forall\, j \in \{H,F\} \,. \tag{4.7}$$

Here, ε measures the elasticity of substitution between firms i within each country. The same equations hold for the rest of the world, with the slight difference that, since the rest of the world's imports from the small open economy are so small, their weighting coefficient α^* is assumed to be negligible. This means that $P^*_{H,t}$, the price index of domestic products in foreign currency, has no influence on the world consumer price index for $\lim_{\alpha^*\to 0}$. This implies $P^*_{F,t} = P^*_t$, where an asterisk denotes the world economy.

The first differences of the logarithms of the price levels are the CPI inflation $\pi_t \equiv \log(P_t) - \log(P_{t-1})$ and the domestic goods (price index) inflation $\pi_{H,t} \equiv \log(P_{H,t}) - \log(P_{H,t-1})$.[4] For the world economy it follows from above that $\pi^*_{F,t} = \pi^*_t$.

[4]Throughout the paper small, Latin letters are used to denote that log-linearization around the steady

Competitive Equilibrium: Firms

A firm's profits are turnover minus total costs, $P_{H,t}(i)Y_t(i) - (1-\tau)W_tN_t(i)$, where the employment subsidy τ lowers the costs of labor. Thus, nominal marginal costs[5] are $MC_t^n = (1-\tau)W_t/A_t$. In the Calvo (1983) staggered price setting scheme, the possibility to reset prices cannot be guaranteed at every period: each period, only the fraction $1-\theta$ of the firms can reset prices.[6] Denoting a newly set price by $\overline{P}_{H,t}(i)$, a representative firm i faces the following maximization problem:[7]

$$\max_{\overline{P}_{H,t}(i)} \sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k}[Y_{t+k}(i)(\overline{P}_{H,t}(i) - MC_{t+k}^n)]\} , \tag{4.8}$$

subject to the demand function

$$Y_{t+k}(i) \leq \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} (C_{H,t+k} + \frac{1}{1-\kappa} C_{H,t+k}^*) \equiv Y_{t+k}^d(\overline{P}_{H,t}). \tag{4.9}$$

Competitive Equilibrium: Governments

Domestic fiscal policy is faced with the following budget constraint:
$T_t = \int_0^1 \tau W_t N_t(i) di$, with T lump sum taxes and τ an employment subsidy. The fiscal authority acts solely to offset the distortion through monopolistic competition. World fiscal policy symmetric, with variables T_t^*, τ^*, W_t^*, $N_t^*(i)$. Monetary policy in the rest of the world is assumed to follow a Taylor-type rule that fully stabilizes its inflation rate and the output gap. For the small open economy, I deviate from GM, who look at the three different monetary policy regimes domestic inflation targeting (DIT), CPI inflation targeting (CIT) and an exchange rate peg. Instead, to make the model more realistic and to alleviate the model's capability to match empirical data, I follow Clarida et al. (1998) and include a Taylor rule (TR):

$$r_t = \overline{rr}_t + \Phi_\pi \pi_{H,t} + \Phi_y(y_t - \bar{y}_t) , \tag{4.10}$$

state has taken place. For the inflation rates given in the text, this steady state can be dropped, as it is zero. More on the steady state is provided in Section 5.3.1 in the appendix.

[5]Observe that nominal total costs $TC_t^n(i) = (1-\tau)W_tN_t(i) = (1-\tau)W_tY_t(i)/A_t$, so that $MC_t^n(i) = \partial TC_t^n(i)/\partial Y_t(i) = (1-\tau)W_t/A_t$.

[6]The assumption is "that each price-setter (or firm) is allowed to change his price whenever a random signal is 'lit up', see Calvo (1983, p. 383).

[7]The maximization problem is derived and explained in Section 5.3.5 in the appendix.

where r is a nominal short-term interest rate, $\overline{rr}$ the natural interest rate, π_H the domestic goods inflation rate, and $\bar{y}_t$ the natural level of output.[8]

Competitive Equilibrium: Trade

There are three exchange rates in this model. The nominal exchange rate is the price of foreign currency in terms of home currency. As in OR, I allow for "iceberg"-type costs of trade in the goods market like transportation costs, tariffs etc. These costs affect the economy in such a way that only a fraction $1-\kappa$ of each good exported arrives at the destination market, whereas the other fraction κ "melts away" in the trade process. As markets are competitive internationally, arbitrage considerations force this effect to show up in cross-border price index relations. For the price of foreign goods, this implies:

$$P^*_{F,t}\mathscr{E}_t = (1-\kappa)P_{F,t}, \tag{4.11}$$

whereas for the price of home goods, these have to sell cheaper abroad:

$$(1-\kappa)P^*_{H,t}\mathscr{E}_t = P_{H,t}\ . \tag{4.12}$$

Log-linearizing (4.11) and (4.12) around the steady state and rearranging results in

$$p_{F,t} = e_t + p^*_{F,t} \tag{4.13}$$

$$p_{H,t} = e_t + p^*_{H,t}, \tag{4.14}$$

where lower bar letters denote log-deviations of the upper bar letters around steady state, which is described in Section 5.3.1 in the appendix. The terms of trade are the price of foreign goods in terms of home goods. In the small open economy, this might read $\mathscr{S}^{\text{soe}}_t = P_{F,t}/P_{H,t}$, whereas for the world economy this is $\mathscr{S}^{\text{world}}_t = P^*_{F,t}/P^*_{H,t}$. Notice, however, that the terms of trade in the last two equations differ by the constant factor $(1-\kappa)^2$, according to Equations (4.11) and (4.12). One could choose either the small open economy's price ratio or the world economy's price ratio for the definition of the terms of trade – or something in between. Following Samuelson (1954), I define intermediate terms of trade:[9]

$$\mathscr{S}_t \equiv (1-\kappa)\frac{P_{F,t}}{P_{H,t}} = \frac{1}{1-\kappa}\frac{P^*_{F,t}}{P^*_{H,t}}. \tag{4.15}$$

[8]The expression "natural" is meant to indicate a situation without nominal frictions.

[9]With this "intermediate" definition, I also make sure that the steady state terms of trade are equal to unity, as it is the case in GM. See also Galí and Monacelli (2005b, Appendix A).

For the log-linear terms of trade,

$$s_t = p_{F,t} - p_{H,t}\ , \tag{4.16}$$

since $p^*_{F,t} = p^*_t$ as $\lim_{\alpha^* \to 0}$. The real exchange rate is the ratio of the two consumer price indices, measured in domestic currency:

$$\mathcal{Q}_t \equiv \mathcal{E}_t P^*_t / P_t. \tag{4.17}$$

In terms of log deviations from steady state, the log real exchange rate $q_t \equiv \log(\mathcal{Q}_t) - \log(\overline{\mathcal{Q}})$ is given as follows:

$$q_t = e_t + p^*_t - p_t\ . \tag{4.18}$$

Because of the producer currency pricing trade costs have no influence on the firms' decisions of price setting. The law of one price obviously holds only in the case of zero trade costs. If domestic goods and foreign goods price indices were equal ($p_{H,t} = p_{F,t}$), α would measure the share of foreign goods' consumption, which could be interpreted as a degree of openness. In this model instead, Section 5.3.1 in the appendix shows that I have a steady state where $\bar{P}_H = (1-\kappa)\bar{P}_F$. The situation around such a steady state can be expressed through log-linearization of (4.6) as

$$p_t = (1-\alpha')p_{H,t} + \alpha' p_{F,t}, \tag{4.19}$$

where $\alpha' \equiv \alpha/[\alpha + (1-\alpha)(1-\kappa)^{1-\eta}]$.[10] This equation, derived in Section 5.3.1 in the appendix, can be combined with Equation (4.16) to obtain the following relationship between domestic CPI and the terms of trade:

$$p_t = p_{H,t} + \alpha' s_t \tag{4.20}$$

Replacing $p_{F,t}$ in Equation (4.16) by Equation (4.13) and plugging the result in (4.18) gives rise to a relationship between the domestic CPI, the terms of trade and the real exchange rate:

$$q_t = (1-\alpha')s_t. \tag{4.21}$$

Nominal net exports are given by

$$P_{H,t} NX_t = P_{H,t} Y_t - P_t C_t. \tag{4.22}$$

[10]Note that $\alpha' = \alpha$ as in GM for $\kappa = 0$.

As Section 5.3.3 of the appendix shows, log-linearizing this equation around the steady state results in

$$nx_t = y_t - \frac{PC}{P_H Y}(c_t + \alpha' s_t), \tag{4.23}$$

where the steady state ratio $\frac{PC}{P_H Y}$ depends on the parameters α, κ, η and σ and equals unity in the case of zero trade costs.

Competitive Equilibrium: Market Clearing

Since there is no possibility to invest in capital, and as the small open economy is negligible for the rest of the world, the foreign country's goods market is cleared if output supply equals its own consumption:

$$Y_t^* = C_t^* \, . \tag{4.24}$$

In the small open economy, output is consumed at home or abroad. However, because a fraction κ of the bundle exported "melts away" in the trade process, consumption abroad is only $1-\kappa$ times the domestic bundle intended for export, $C_{H,t}^* = (1-\kappa)(Y_t - C_{H,t})$. Hence, in the small open economy goods market clearing is given by

$$Y_t = C_{H,t} + \frac{1}{1-\kappa} C_{H,t}^* \, . \tag{4.25}$$

In the labor markets, firms set wages so that their demand of labor is supplied by the domestic agents. The international asset market is cleared as the nominal portfolio is in zero net supply. On the currency market, each countries' central bank supplies the amount of currency that is demanded.

Definition 4. *Given policy rules for R_t, an equilibrium is an allocation $\{D_t, C_t, (C_{j,t})_{j\in\{H,F\}}, (C_{i,j,t})_{i\in[0,1]}, L_t, Y_t, (Y_{j,t})_{j\in\{H,F\}}, (Y_{i,j,t})_{i\in[0,1]}\}_{t=0}^{\infty}$ and a price system $\{W_t, P_t, (P_{j,t})_{j\in\{H,F\}}, (P_{i,j,t})_{i\in[0,1]}\}_{t=0}^{\infty}$, such that*

1. *given prices, the allocation maximizes the utility of the household,*
2. *given prices and the demand function for $Y_{i,j,t}$, the allocation maximizes the profits of the firms, subject to the Calvo-sticky prices,*
3. *markets clear,*
4. *the policy rule is consistent with allocation and prices.*

4.2.3 Analysis

Analysis: Households

The expenditures of the representative household are distributed optimally between all firms of a country as well as between home country and the rest of the world in the aggregate. The allocations will be:

$$C_{j,t}(i) = \left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\varepsilon} C_{j,t} \quad \forall j \in \{H,F\} \tag{4.26}$$

within each country, and for total consumption:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \; . \tag{4.27}$$

Maximizing the household's utility function leads to a standard intratemporal equation linking marginal utilities of labor and consumption to the real wage:

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{4.28}$$

and a typical Euler equation:

$$\beta R_t E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right) = 1 \; . \tag{4.29}$$

Log-linearization yields

$$w_t - p_t = \sigma c_t + \varphi n_t \quad \text{and} \quad c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\}). \tag{4.30}$$

As shown in the appendix Section 5.3.4, Equation (4.29) and its world analog[11] can be combined and iterated to get a relation for consumption in both economies:

$$C_t = \vartheta C_t^{*} \mathcal{Q}_t^{\frac{1}{\sigma}} \; , \tag{4.31}$$

where the parameter ϑ depends on initial conditions regarding the relative size of the small open economy.[12] In log-deviations and using Equation (4.21), the last equation

[11] Under complete markets for nominal state contingent securities (see Monacelli 2005), $\beta R_t E_t[(\frac{C^*_{t+1}}{C^*_t})^{-\sigma}(\frac{P^*_t}{P^*_{t+1}})(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}})] = 1$ holds.

[12] It is assumed that the initial distribution of wealth fulfills $\vartheta = \frac{\alpha^*}{\alpha}$, i.e. equals the ratio of the two economies' import valuations.

becomes

$$c_t = c_t^* + \left(\frac{1-\alpha'}{\sigma}\right) s_t \, . \tag{4.32}$$

Analysis: Firms

Aggregation of individual firms' production functions and log-linearizing around the steady state yields the (log) supply of output

$$y_t = n_t + a_t \, . \tag{4.33}$$

In every period, firm i has a probability of $(1-\theta)$ that it is allowed to adjust its price. If this is the case in period t, and as each firm has market power, it sets its new price $\overline{P}_{H,t}$ with a markup over marginal costs so that for the expected duration of that price the present discounted value of its expected earnings is maximized. Given the maximization problem of Equations (4.8) and (4.9) and as shown in appendix Section 5.3.5, the log-linear price setting rule is

$$\overline{p}_{H,t} = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\widehat{mc}_{t+k}^n\} \, , \tag{4.34}$$

where $\overline{p}_{H,t}$ is the newly set price in period t and $\widehat{mc}_t^n$ is the log-deviation of nominal marginal costs around the steady state. As appendix Section 5.3.6 shows, the inflation dynamics in the small open economy and in the world economy are given by

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda(\widehat{mc}_t) \quad \text{and} \quad \pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \lambda(\widehat{mc}_t^*) \, , \tag{4.35}$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

Analysis: Governments

Both fiscal policy authorities set their employment subsidy to offset monopolistic distortion. For reasons of comparability with GM I rely on their values,[13] i. e. for the small open economy

$$\tau = 1 - \frac{\varepsilon - 1}{(1-\alpha)\varepsilon} \quad \text{and} \quad \tau^* = \frac{1}{\varepsilon} \tag{4.36}$$

[13] GM derive these values under the special case in which $\sigma = \eta = 1$ holds. See Galí and Monacelli (2002, pp. 22ff.).

for the world economy, where the α^*-term drops as the degree of openness there is essentially zero.

Monetary policy in the world economy leads to a fully stable world output gap and world inflation rate, so that I can set both variables to zero:
$\tilde{y}_t^* = \pi_t^* = 0$.
This drives the world interest rate to its natural level, so that I get

$$r_t^* = -\sigma(1-\rho_a^*)\frac{1+\varphi}{\sigma+\varphi}a_t^* \ . \tag{4.37}$$

The authority for monetary policy in the small open economy follows the Taylor rule given in Equation (4.10). Alternatively, I also analyze a strict domestic inflation targeting (DIT) policy, a domestic inflation targeting rule (DITR), a CPI targeting rule (CITR) and an exchange rate peg (PEG).

Analysis: Canonical Representation

The model can be written in four equations, a Phillips curve and a dynamic IS curve for both the small open and the world economy. Denoting a variable's deviation from its natural level that would pertain in a flexible price world by an upper tilde, the equations are:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \Phi_{NKPC}\tilde{y}_t \tag{4.38}$$

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \Phi_{NKPC^*}\tilde{y}_t^* \tag{4.39}$$

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{\omega}{\sigma}(r_t - E_t\{\pi_{H,t+1}\} - \overline{rr}'_t) \tag{4.40}$$

$$\tilde{y}_t^* = E_t\{\tilde{y}_{t+1}^*\} - \frac{1}{\sigma}(r_t^* - E_t\{\pi_{t+1}^*\} - \overline{rr}_t^*) \ , \tag{4.41}$$

where $\Phi_{NKPC} \equiv \lambda\left(\frac{\sigma}{\omega}+\varphi\right)$, $\omega \equiv \sigma\eta + (1-\sigma\eta)(1-\alpha')\left(1-\frac{\alpha}{1-\kappa}\Phi_{SS2}^{-1}\right)$ with α' defined after Equation (4.19) and Φ_{SS2} after Equation (4.23), and $\Phi_{NKPC^*} \equiv \lambda(\sigma+\varphi)$. The $\overline{rr}$-terms are the natural expected rates of interest in the small open and the world economy, respectively, which would prevail under completely flexible prices. They are given by

$$\overline{rr}_t \equiv -\frac{\sigma(1+\varphi)(1-\rho_a)}{\sigma+\omega\varphi}a_t - \varphi\frac{\sigma(1-\omega)}{\sigma+\omega\varphi}E_t\{\Delta y_{t+1}^*\} \tag{4.42}$$

and

$$\overline{rr}_t^* \equiv -\sigma(1-\rho_a^*)\frac{1+\varphi}{\sigma+\varphi}a_t^* . \tag{4.43}$$

A derivation of these equations is given in appendix Section 5.3.7. Together with rules for monetary policies and the exogenous stochastic processes, the model is complete.

4.3 Puzzles and Deduced Moments

This section briefly states the six puzzling data observations, as collected by OR. It then focuses on the specific moments of the data that may be used to evaluate the corresponding moments of the model and thereby the model's fit. In choosing these data moments I often allow for a wide range of values. This is the consequence of the existing variability in observation moments across time and countries.

4.3.1 Home Bias in Trade (Puzzle 1)

In an Arrow-Debreu world of complete international markets without any barriers to trade, an equal amount of products is traded across international and intra-national borders, so that borders do not matter for trade. In reality, we see that there is significantly less trade across international borders, i.e., domestic products are preferred. This is pointed out by e. g. McCallum (1995) for the example of the U.S. versus Canada. McCallum found 22 times less trade across the border than across interstate borders in Canada or in the U.S. In a more careful study, Anderson and van Wincoop (2003) argue that borders reduce trade between industrialized countries by 29 percent or, in the case of U.S. - Canadian trade, by 44 percent.

OR propose to use the ratio of domestic consumption expenditure on home goods to domestic consumption expenditure on imported goods as moment for the home bias in trade. They argue that 4.2 is a reasonable value for OECD countries. This implies a home share in consumption of about 80 percent. Clearly, this number depends on the size of the country considered: the smaller the country, the fewer goods are produced domestically, and the lower the number gets. As a starting point, I take values above unity as consistent with a home bias. To rule out too distinct a bias, I set an upper limit of 19, implying a home share in consumption of 95 percent. Hence, my first moment is the steady state ratio

$$\mathrm{P1} = \frac{P_H C_H}{P_F C_F} = \frac{1-\alpha}{\alpha}(1-\kappa)^{1-\eta} \in [1;19], \tag{4.44}$$

depending only on home bias parameter α, trade costs κ and international substitution elasticity η, according to Equations (4.27) and (4.15), evaluated at the steady state.

4.3.2 Feldstein-Horioka (Puzzle 2)

If one supposes that capital can move freely across countries and people are free to invest their money wherever they want, one would suspect that rising savings in one economy do not necessarily imply rising investments in the same country. If conditions for investment are temporarily better abroad, the savings should all be directed to foreign countries, leaving investments in the home country constant or reducing them. With this in mind one would expect a low correlation between savings and investment in open economies with free capital movements. Instead, the data shows a high positive correlation: Feldstein and Horioka (1980) found a coefficient of 0.89 for 16 OECD countries between 1960 and 1974. A regression for a 22 OECD country sample between 1982-91 by Obstfeld and Rogoff (1996, p. 162) results in a coefficient of 0.62, while the latest regression by the same authors (Obstfeld and Rogoff 2000*b*, Table 1) for the 24 OECD countries between 1990-97 yields 0.60. Although there is a decreasing trend, the absolute value of the correlation coefficient is still large.

To evaluate where the model's savings are invested, one has to solve for the country portfolios. Given that I use a log-linear approximation to find the model solution, this is not an easy task, for two reasons, as pointed out by Devereux and Sutherland (2007, p.9): "Firstly, the equilibrium portfolio is indeterminate in a first-order approximation of the model. And secondly, the equilibrium portfolio is indeterminate in the non-stochastic steady state." Recently, researchers have drawn their attention to this problem and have come up with different solution approaches, e.g. Coeurdacier and Gourinchas (2008), Coeurdacier (2009), Devereux and Sutherland (2007) and Engel and Matsumoto (2008).[14] One finding of these papers highlighted in Coeurdacier and Gourinchas (2008) is that in a complete markets model, "the equilibrium equity portfolios are extremely sensitive to the values of preference parameters. Whether the coefficient of relative risk aversion is smaller, bigger than or equal to unity, whether domestic and foreign goods are substitute or complements, equity portfolios can exhibit home, foreign, or no bias. In other words, this class of models predict delivers equity portfolios that are *unstable*." Because of this, and because of comparability between my results and those derived in Obstfeld and Rogoff (2000*b*), in the following I stick to the approach OR take to address this puzzle. They built a stylized model to show that

[14]A lucid summary of the recent developments is given in Obstfeld (2007).

"countries running current account surpluses should have lower real interest rates than countries running deficits."[15] This implies a negative correlation between net exports nx_t and the domestic real interest rate $r_t - \pi_t$. So I take as the second moment

$$\mathrm{P2} = \mathrm{Corr}(nx_t, r_t - \pi_t) \in [-1;0]. \tag{4.45}$$

Of course, one may cast doubts on this correlation as adequate translation of the Feldstein-Horioka puzzle, and indeed Jeanne (2000) has raised concerns against this approach. But for the current study, I leave this issue unresolved and take the moment at face value.

4.3.3 Home Bias in Equity Portfolio (Puzzle 3)

In 2005, Canadians held about 76 percent of their equity wealth in their domestic stock market. However, the Canadian equity market capitalization accounted for less than four percent of the world equity market capitalization. In a world of complete risk diversification, this pronounced home bias is difficult to explain. The average home bias across 20 OECD countries is 70 percent, ranging from 31 percent for the Netherlands to above 90 percent for countries like Japan, Greece or Russia.[16] In my model, there is free and costless trade in a complete set of state-contingent Arrow-Debreu securities. Under complete markets, consumption shares are equal to shares in world wealth. Obstfeld and Rogoff (1996, Section 5.3) show that (given zero trade costs) these shares are also equal to portfolio shares. For the special case in which $\sigma = 1/\eta$ holds, the Arrow-Debreu allocation is identical to a world where trade is only in equity shares.[17] In that case one can thus evaluate home bias in equity portfolios directly. For the more general case where $\sigma \neq 1/\eta$, OR show that consumption shares are nonetheless relatively constant over a wide range of parameter combinations and are thus a good approximation to equity portfolio shares.[18] Hence, I follow OR and rely on steady state consumption shares as an indicator for equity portfolio shares. I define the small open economy's steady state home bias equivalently to the portfolio home bias definition given in Coeurdacier and Gourinchas (2008):[19] Home bias is given as one minus the share of foreign equities (consumption) in the small open economy's equity holdings

[15] See Obstfeld and Rogoff (2000*b*, p.358) and Table 3 therein for empirical evidence.

[16] Data from Sercu and Vanpee (2008), as reprinted in Coeurdacier and Gourinchas (2008).

[17] See OR and Obstfeld and Rogoff (1996, Sections 5.2 and 5.3).

[18] See Obstfeld and Rogoff (2000*b*, pp. 363 and Table 4). Obstfeld (2007) emends an approximation error, which nonetheless does not overturn the general picture.

[19] The last page shows a reprint of the 2007 version of Sercu and Vanpee (2008). The published version avoids the term "home bias".

(total consumption), divided by the share of foreign equities (consumption in the rest of the world) in the total market portfolio (overall consumption). By definition the home bias is zero in case the share of domestic equities (consumption) in the small open economy is equal to the share of domestic equities (consumption) in the total world portfolio (consumption). Hence, my third moment is

$$P3 = 1 - \frac{\frac{C_F}{C}}{\frac{C^*}{C+C^*}} = 1 - (1 + \vartheta \Phi_{PHP}^{\frac{1}{\sigma}}) \alpha \Phi_{PFP}^{-\eta} \in [0.32; 0.92]. \quad (4.46)$$

Notice that I have used Equations (4.31) and (4.27) at the steady state to rephrase the equation. One can see that the moment depends on the parameters α, η, κ and α^* only, where the last parameter is assumed to be fixed.

4.3.4 Low International Consumption Correlation (Puzzle 4)

If risks were pooled internationally, changes in consumption would be perfectly correlated across countries to hedge against country specific risk. However, in the real world this is not the case. Despite the intuitive relative consumption smoothing argument, consumption is even less correlated internationally than is output: compared to the “world" analog, the correlation of consumption growth in the OECD countries lies somewhere between 0.27 for Italy and 0.63 for Germany, with an average of 0.43. At the same time, output growth correlations are nearly always higher, between 0.42 for Japan and 0.70 for Canada and Germany, with an average of 0.52.[20] Backus et al. (1995, Tables 1 and 2) report correlations relative to the U.S. instead of a “world" analog. Hence, they have slightly different numbers, but generally the same findings. Moreover, they find productivity[21] to be internationally less correlated than output. They call this puzzle “the consumption/output/productivity anomaly, or the quantity anomaly".[22] I choose the ratio of consumption to output correlations as my fourth moment, which is between about 0.5 for Italy and about 1 for the U.K.:

$$P4 = \frac{\mathrm{Corr}(c_t, c_t^*)}{\mathrm{Corr}(y_t, y_t^*)} \in [0.5; 1]. \quad (4.47)$$

[20] Obstfeld and Rogoff (1996, p. 291), data from Penn World Tables for the period 1973 to 1993. The “world" analog means 35 benchmark countries.

[21] Productivity is measured by the Solow residual z of a standard Cobb-Douglas production function $Y_t = Z_t K_t^{\theta} N_t^{1-\theta}$.

[22] Backus et al. (1995, p. 343).

4.3.5 Purchasing Power Parity (Puzzle 5)

Rogoff (1996) phrases the purchasing power parity puzzle question as follows: “How can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out?"[23] The standard deviation of the real exchange rate typically amounts to about eight percent.[24] The autocorrelation of the real exchange rate $\text{Corr}(q_t, q_{t-1})$ is about 0.83.[25] As this puzzle has two dimensions, I collect two data moments based on Chari et al. (2002):

$$\text{P51} = \text{Std}(q_t) = 7.52 \tag{4.48}$$

$$\text{P52} = \text{Corr}(q_t, q_{t-1}) = 0.83. \tag{4.49}$$

4.3.6 Exchange Rate Disconnect (Puzzle 6)

Another fact concerning the real, but also to the nominal exchange rate is the missing of a strong connection to any other macroeconomic variable. This feature can be examined from two points of view: *a*) a connection could be seen if the high volatility of exchange rates would have an effect on the volatility of some other macroeconomic variable. In this respect, the disconnect shows up in a situation in which, “while exchange rate volatility is ultimately tied to volatility in the fundamental shocks to the economy, the exchange rate can display extremely high volatility without any implications for the volatility of other macroeconomic variables."[26] As Flood and Rose (1995) show, moving from floating to fixed exchange rates or into the other direction does not influence the volatility of other macroeconomic variables. *b*) The disconnect is also a question of correlations between the exchange rate and other variables such as output or prices. Kollmann (2001, p. 254) reports correlations with domestic GDP between -0.21 and 0.15 for Japanese, German and U.K. post-Bretton Woods data, on average -0.07 for the nominal and -0.01 for the real exchange rate. As for the previous puzzle, I select two moments: first, the standard deviation of the real U.S. $ exchange rate rela-

[23] Rogoff (1996, p. 647).

[24] Chari et al. (2002, Table 2) report 7.52 percent for quarterly, logged, Hodrick-Prescott (HP)-filtered European post-Bretton Woods real exchange rates relative to the U.S. Dollar, Kollmann (2001, p. 254) reports 8.89 percent for an average of Germany, Japan and the U.K. versus the U.S.

[25] Chari et al. (2002, Table 1) report values between 0.77 and 0.86 for quarterly, logged, Hodrick-Prescott (HP)-filtered European post-Bretton Woods data relative to the U.S. Dollar, with an average of 0.83. Kollmann (2001, p. 254) comes to a value of 0.78 for a slightly shorter time span of data for Japan, Germany and the U.K.

[26] Devereux and Engel (2002, p. 4).

tive to that of real GDP, which is 4.36 percent, according to Chari et al. (2002).[27] Second, the contemporaneous correlation between the real U.S. $ exchange rate and real GDP, which Chari et al. (2002) report to be 0.08.[28]

$$\text{P61} = \text{Std}(q_t)/\text{Std}(y_t) = 4.36 \tag{4.50}$$

$$\text{P62} = \text{Corr}(q_t, y_t) = 0.08. \tag{4.51}$$

While puzzles 1 and 3 follow immediately from the model's steady state, the remaining moments are obtained from simulations of the model. I average the moments of 500 simulations of 100 periods length.

4.4 Parametrization

For the specification of parameter values I will make use of three different procedures. In a first step, I use calibration to obtain values for those parameters that have (a) agreed upon values in the literature and (b) no significant effect on the model outcome with respect to the six puzzles. In a second step, I identify a set of parameter values via estimation.This procedure is applied to parameters that have a close relationship to observable data, like exogenous processes and the Taylor rule. The third step is choosing the remaining parameter values to minimize the distance between simulated moments from the model and the moments implied by the “six puzzles". This procedure is applied in Jermann (1998) to “maximize the model's ability to match a set of moments of interest"[29]. A textbook treatment under the label *Simulated Method of Moments Estimation* (SMM) is given in Canova (2007, Section 5.5.2).

I use data for Canada versus the U.S. for two reasons. First, because of its relative size and proximity to the U.S., Canada is “a prototype small open economy".[30] Not only is Canada a relatively small country, it also trades mainly with the U.S.[31] so that the assumption of the U.S. as the rest of the world seems especially plausible. Second, Galí and Monacelli (2005b) use Canadian data for their numerical analysis. So it seems fair to stick to the same data when putting the model to test. The dataset used for the analysis is the one built by Chari, Kehoe, and McGrattan (2002), added by central bank

[27]See Table 5 in Chari et al. (2002). Kollmann (2001, Table 1) reports $\frac{8.89}{1.52} = 5.85$ percent.
[28]See Table 6 in Chari et al. (2002). Kollmann (2001, Table 1) reports -0.01.
[29]Jermann (1998, p. 264).
[30]Galí and Monacelli (2005b, p. 723).
[31]According to `en.wikipedia.org`, about 80 percent of Canadian exports go to and about two thirds of Canadian imports come from the U.S.

short term interest rates obtained from IFS. It contains quarterly macroeconomic data for Canada and the U.S. from 1973:1 till 2000:1, obtained from the IMF's IFS and the OECD.[32] The data are seasonally adjusted, in logs, and HP-filtered. The series contain real GDP, consumption, net exports, CPI price level, PPI price level, nominal and real exchange rate, terms of trade and employment. Series for technology are obtained by use of Equation (4.33) and its world analog.[33]

4.4.1 Calibrated Parameter Values

Results for the first procedure (calibration) are given in column two of Table 4.1. Mostly, the values were chosen in accordance with those of the GM model. The (quarterly) discount factor β is set to 0.987 according to Cooley and Prescott (1995, p. 21). The net steady state markup μ of roughly 20 percent over marginal costs is consistent with the findings of Rotemberg and Woodford (1995, pp. 260-261) as well as Schmitt-Grohé and Uribe (2004, p. 11). With μ fixed I have already set the elasticity of substitution between different firms within a country ε to be six, through $\mu = \log(\varepsilon) - \log(\varepsilon - 1)$ from Section 5.3.7 in the appendix. The labor supply elasticity $1/\varphi$ is fixed at $1/\varphi = 1/3$, like in GM. Benigno (2004) proposes a value of 0.67, whereas Blanchard and Fischer (1989) report a low value between 0 and 0.45.[34] Yun (1996) calibrates his model with $1/\varphi = 1/4$. I also tested values between zero and unity and found that the model's performance is not affected. Finally, the degree of openness parameter for the world economy α^* has to be fixed close to zero to maintain the small open economy assumption.

4.4.2 Estimated Parameter Values

The second procedure was applied for the Taylor rule (TR). Again, results are given in Table 4.1, columns three and four. For t estimation of the Taylor rule for Canada I follow the example of Clarida et al. (1998) and use the generalized method of moments (GMM). Instruments are eight lags of inflation, output gap and interest rate ($R^2 = 0.82$, standard errors in parentheses).

$$r_t = \underset{(0.02)}{0.90} * r_{t-1} + (1 - 0.9) * (\underset{(0.15)}{2.20} * \pi_{t+1} + \underset{(0.83)}{2.43} * (y_t - \bar{y}_t)) + \varepsilon_t^M . \tag{4.52}$$

[32]The original dataset contains data 17 OECD countries and a longer sample period for most series, which allows for an extended analysis in future work.

[33]This results in the standard correlation pattern given e.g. in Uhlig (2003).

[34]See Blanchard and Fischer (1989), Chapters 7 and 8, especially pp. 338-342 and 388.

For the estimation of the stochastic processes I rely on Galí and Monacelli (2005b). They assume AR(1) processes for log Canadian labor productivity and log U.S. GDP and obtain

$$a_t = \underset{(0.06)}{0.66} a_{t-1} + \epsilon_t, \qquad \text{Std}(\epsilon) = 0.0071 \tag{4.53}$$

$$y^*_t = \underset{(0.04)}{0.86} y^*_{t-1} + \epsilon^*_t, \quad \text{Std}(\epsilon^*) = 0.0078 \tag{4.54}$$

with a correlation between the two shocks of 0.3. Standard errors are given in parentheses. It is clear that the international correlation of productivity shocks will have an influence on the puzzle outcomes. Especially the international consumption correlation and the real exchange rate correlation would be significantly affected if I took this parameter as free in my minimization procedure laid out below. Nonetheless, I abstain from making use of this opportunity as I regard this parameter to be given by the data.

4.4.3 Simulated Method of Moments Parameter Values

Applying the third procedure, I single out five parameters that mainly influence the model's features relative to the six puzzles or, in the case of price stickiness, are key to this class of models. These are the international substitution elasticity η, the constant of relative risk aversion σ, the small open economy's openness parameter α, the Calvo price stickiness parameter for both economies $\theta = \theta^*$ and the trade costs parameter κ. Let Θ_1 be the vector of these five model parameters: $\Theta_1 = [\eta, \sigma, \alpha, \theta, \kappa]'$. I choose Θ_1 in order to minimize

$$\Im = [\Theta_2 - f(\Theta_1)]'\Omega[\Theta_2 - f(\Theta_1)], \tag{4.55}$$

where $\Theta_2 = [P1, P2, P3, P4, P51, P52, P61, P62]'$ is the vector of moments to be matched, given by equations (4.44) to (4.51). $f(\Theta_1)$ is a 8×1 vector which contains the corresponding moments generated by the model. The weighting matrix Ω is chosen as a diagonal matrix with the inverse of each data mean as the diagonal elements. Since many of the data moments are given in target ranges, the expression $\Theta_2 - f(\Theta_1)$ is not trivial. Following Uhlig (2004), I allow for these ranges by combining maximum and minimum functions:

$$\Theta_2 - f(\Theta_1) = \begin{bmatrix} \min(19 - f(\Theta_1)_1, 0) + \max(1 - f(\Theta_1)_1, 0) \\ \min(0 - f(\Theta_1)_2, 0) + \max(-1 - f(\Theta_1)_2, 0) \\ \min(0.92 - f(\Theta_1)_3, 0) + \max(0.32 - f(\Theta_1)_3, 0) \\ \min(1 - f(\Theta_1)_4, 0) + \max(0.5 - f(\Theta_1)_4, 0) \\ 7.52 - f(\Theta_1)_5 \\ 0.83 - f(\Theta_1)_6 \\ 4.36 - f(\Theta_1)_7 \\ 0.08 - f(\Theta_1)_8 \end{bmatrix}. \tag{4.56}$$

For the minimization process, the model solution has to be calculated. This is done using standard techniques, as explained in Uhlig (1999). To minimize the criterion function $\Im$, I furthermore need to set starting values and boundaries to the parameters in Θ_1.

The elasticity of substitution between domestic and foreign goods η typically takes values between unity, as in Galí and Monacelli (2005b) and something as high as 20, as Obstfeld and Rogoff (2000*b*) say. In between lie $\eta = 1.5$ as in Backus et al. (1995, pp. 346-347.) and the OR benchmark of $\eta = 6$. The higher the substitutability between domestic and foreign goods, the bigger the home biases get. But there is a theoretical qualification to this. The elasticity of substitution between different domestic goods ε is set to six, in order to allow for a steady state markup of 20 percent above marginal costs. It seems unrealistic that substitutability is much higher internationally than intranationally. Engel (2000) raises exactly this question at the end of his comment on the “Six Puzzles"; he proposes the intranational elasticity to be twice as high as the international. I follow his suggestion and restrict η to be between 1 and 12. As a a starting value, I set $\eta = 3$.

The risk aversion parameter σ, also the inverse of the intertemporal rate of substitution, is difficult to determine: GM and Yun (1996) use $\sigma = 1$, implying log utility of consumption. Erceg, Henderson, and Levine (2000, p. 299) use 1.5 for σ, Cochrane calls values between one and two standard,[35] Chari, Kehoe, and McGrattan (2002) choose a high value of $\sigma = 5$ and argue that this is needed to obtain volatile exchange rates. Like GM, I use $\sigma = 1$ as my starting value and allow it to be between 0.2 and as much as 10, which is also the upper bound in Anderson and van Wincoop (2004).

The degree of openness parameter α should be between zero and unity, where one

[35] Cochrane (1997, p. 15). The asset pricing literature yields for even higher values to explain the equity premium puzzle.

half implies no home bias and more than one half is a bias towards foreign goods. GM choose $\alpha = 0.4$ as their baseline value to match the import to GDP ratio for Canada. I follow them with my starting value and set the boundaries to zero and 0.9, where the upper boundary implies a bias towards foreign goods. This might be especially reasonable for very small countries which produce only a restricted subset of all goods.

The Calvo sticky price parameter $\theta = \theta^*$, assumed to be identical across countries, is typically set to 0.75, implying an average price duration of four quarters, $\frac{1}{1-\theta} = 4$. This is also my starting value. In the SMM estimation, I choose θ from the interval $[0.0, 0.9]$, implying price changes between every quarter and every 10th quarter.

Finally, the trade costs' starting value is set to 25 percent, the value OR choose as their "baseline". Midrigan (2007) chooses a distribution of trade costs that replicates moments of certain export shares. He comes up with trade costs between 2 percent and 48 percent, with a mean of 20 percent. Relative to the sources reported in OR, 20 or 25 percent are high, but taking into account that about a half of total output is nontraded, the number might become more reasonable. Anderson and van Wincoop (2004) report a 170 percent tax equivalent of trade costs. This number breaks down into 21 percent transportation costs, 44 precent border related trade barriers, and 55 percent retail and wholesale distribution costs. Of course, "iceberg" trade costs cannot be bigger than unity, as unit trade costs lead to autarky of the two then closed economies. Given the degree of uncertainty about this parameter, I hardly restrict the SMM estimation using the interval $[0.0; 0.9]$.

Boundaries and starting values for the parameters in Θ_1 are given in columns four and two of Table 4.1. The resulting estimates are given in column five of the same table.

Table 4.1: Benchmark Parameter Values

Parameter	Calibration	Estimation	SMM Range	SMM	Explanation
Preferences					
β	0.987	–	–	–	Discount factor
η	3.00	–	[1.0; 12]	1.0	Elasticity of substitution between domestic and foreign goods
ε	6.00	–	–		Elasticity of substitution among goods within each category
σ	1.00	–	[0.2; 10]	3.15	Constant of relative risk aversion
φ	3.00	–	–	–	Inverse of labor supply elasticity
α	0.40	–	[0.0; 0.9]	0.05	Degree of openness of the small open economy
α^*	0.001	–	–	–	Degree of openness of the world economy
Technology					
$\theta = \theta^*$	0.75	–	[0.0; 0.9]	0.78	Percentage of firms that cannot (re)set prices in period t
μ	0.182	–	–	–	Log of the gross steady state markup
κ	0.25	–	[0.0; 0.9]	0.39	Trade costs
Monetary Policy					
ρ^{TR}	–	0.90 (0.02)	–	–	Degree of interest rate smoothing
β^{TR}	–	2.20 (0.15)	–	–	Coefficient on next period inflation
γ^{TR}	–	2.43 (0.83)	–	–	Coefficient on output gap
Processes					
σ_{ϵ}	–	0.0071 (–)	–	–	Standard deviation of domestic productivity shock
σ_{ϵ^*}	–	0.0078 (–)	–	–	Standard deviation of world GDP shock
ρ_a	–	0.66 (0.06)	–	–	Autocorrelation of domestic productivity AR(1) process
ρ_y^*	–	0.86 (0.04)	–	–	Autocorrelation of world GDP AR(1) process
ρ_{a,y^*}	–	0.30 (–)	–	–	Cross-correlation of productivity shocks

Notes: Column 2 includes calibrated values as well as the starting values for the SMM estimation, column 3 has standard errors in parentheses, column 4 shows the allowed values for the simulated method of moments estimation and column 5 gives the SMM estimates.

4.5 Results

As the title of this paper might suggest, the results of this model are not too bad. Table 4.2 reports how the thus parameterized model performs against the six puzzles.

Table 4.2: Baseline Results for the Taylor Rule Model

Criterion	Moment	Value	Lower	Data	Upper
Puzzle 1	$P_H C_H / (P_F C_F)$	19.36	1		19
Puzzle 2	$\mathrm{Corr}(nx_t, r_t - \pi_t)$	-0.48	-1		0
Puzzle 3	$1 - \frac{C_F/C}{C^*/(C+C^*)}$	0.97	0.32		0.92
Puzzle 4	$\mathrm{Corr}(c_t, c_t^*)/\mathrm{Corr}(y_t, y_t^*)$	2.83	0.5		1
Puzzle 51	$\mathrm{Std}(q_t)$	1.97		7.52	
Puzzle 52	$\mathrm{Corr}(q_t, q_{t-1})$	0.61		0.83	
Puzzle 61	$\mathrm{Std}(q_t)/\mathrm{Std}(y_t)$	3.13		4.36	
Puzzle 62	$\mathrm{Corr}(q_t, y_t)$	0.63		0.08	

Notes: The baseline results use the parametrization given in Table 4.1. In particular, $\kappa = 0.39$, $\theta = 0.78$, $\alpha = 0.05$, $\eta = 1$ and $\sigma = 3.15$. “Data" refers to the target ranges or values discussed in Section 4.3.

We see that with sizeable trade costs of close to 40 percent and a small degree of openness parameter, implying a steady state import share of only five percent, the model is able to replicate strong home biases in consumption and in equity portfolio. These biases are slightly above what is observed for typical small OECD countries, but not by much. Puzzle 2 in its translated form is nicely replicated: The correlation between net exports and the real interest rate is right in the range of what OR estimated for OECD countries. A high relative risk aversion of more than three, low international substitutability and a small degree of openness lead to volatile real exchange rates. This is in accordance with the argument in Hau (2002) that less open economies experience a higher exchange rate volatility. Compared to the model results with calibrated parameter values, the number for real exchange rate volatility is extraordinarily big: Nonetheless, the volatility is not as big as in the data, both per se and relative to GDP volatility. With respect to the correlation pattern of the real exchange rate the findings are mixed: The autocorrelation of the real exchange rate is a bit low in the model, the correlation with GDP is too big. The perhaps worst outcome concerns the consumption correlation puzzle. The ratio of correlations is 2.83, which is way above the expected value of less than one. This ratio is the result of an international output correlation of only 0.14, whereas the international correlation of consumption is 0.47. Though the data does not provide a very clear pattern, this combination is not realistic.

4.5.1 Do Trade Costs Improve the Model's Fit?

The original model of GM does not include trade costs. On the other hand, OR argue that "the effects of home bias in preferences [...] can be isomorphic to the effects of trade costs".[36] So a natural question is whether or not a model with zero trade costs or a model with no home bias can fare equally well. Results to this are reported in Table 4.3.

Table 4.3: Comparison of Results:
Trade Costs and Degree of Home Bias Parameter in the Trade Costs Model

Criterion	Data	Baseline	$\kappa = 0$	$\alpha = .5$	κ=0, α=.5	$\kappa = .9999$
κ	–	0.39	0	0.26	0	0.9999
θ	–	0.78	0.75	0.75	0.75	0.71
α	–	0.05	0.40	0.5	0.5	0.56
η	–	1.00	3.00	3.00	3.00	1.30
σ	–	3.15	1.00	1.00	1.00	0.73
Puzzle 1	$[1;19]$	19.36	1.50	1.84	1.00	13.45
Puzzle 2	$[-1;0]$	-0.48	-0.57	-0.43	-0.33	-0.10
Puzzle 3	$[.32;.92]$	0.97	0.60	0.70	0.50	1.00
Puzzle 4	$[0.5;1]$	2.83	9.13	9.55	10.58	1.14
Puzzle 51	7.52	1.97	0.33	0.34	0.25	0.81
Puzzle 52	0.83	0.61	0.62	0.62	0.62	0.62
Puzzle 61	4.36	3.13	0.37	0.39	0.28	0.97
Puzzle 62	0.08	0.63	0.63	0.63	0.64	0.48
min $\Im$	–	36.10	134.09	140.76	161.49	56.71

As column three of this table shows, the zero trade costs model does not leave out a lot in terms of the correlation between net exports and the real interest rate and in terms of the correlation pattern of real exchange rates. Also, the home bias puzzles can be addressed without relying on trade costs. But the volatility of the real exchange rate is significantly smaller in a model without trade costs. This is an aspect in favor of OR's idea. But notice that the estimation process did not deviate from the parameters' starting values, which may indicate some estimation deficiency. Column four of Table 4.3 shows the case without the home bias in preferences or degree of openness parameter, i.e. $\alpha = 0.5$. The result here is very much comparable to the one obtained in a model without trade costs. Hence, the isomorphic effects of the two parameters are shown here. The case of excluding both trade costs and openness parameter is depicted in

[36] See OR, p. 348.

column five. Here, exchange rate volatility is especially difficult to obtain. As the comparison shows, the combination of trade costs and openness parameter can do a lot in this respect. Finally, the last column shows the estimation outcome if trade costs are fixed to a prohibitively high number $\kappa = 0.9999$, implying that virtually nothing of an exported good arrives at the destination market. This was done just for theoretical considerations. In this case, there is an offsetting foreign bias in consumption, as well as high intertemporal substitutability. While relative consumption correlation ($P4$) is decreased significantly, the outcome on the real exchange rate volatility dimension ($P51$ and $P61$) is worse than in the baseline model.

4.5.2 Alternative Monetary Policy Rules

In this section, I briefly check whether or not the previous results hinge on the estimated monetary policy rule. My deviations from this rule are along the suggestions in Galí and Monacelli (2005b). In particular, I investigate four different monetary policies:

1. Strict domestic inflation targeting (DIT), which GM show to be optimal from a welfare perspective under certain parameter restrictions. This rule can be written as follows:
$$r_t = \overline{rr}_t + \Phi_\pi \pi_{H,t} + \Phi_y \tilde{y}_t, \tag{4.57}$$
where the last two summands are only added to circumvent indeterminacy, as explained in Galí and Monacelli (2005b).

2. A domestic inflation targeting rule (DITR), which relates the domestic short-term nominal interest rate only to the domestic inflation rate,
$$r_t = \Phi_\pi \pi_{H,t}. \tag{4.58}$$

3. A CPI inflation targeting rule (CITR), as given by
$$r_t = \Phi_\pi \pi_t. \tag{4.59}$$

4. And finally an exchange rate peg (PEG) that fixes the domestic nominal interest rate to its world analog,
$$r_t = r_t^*. \tag{4.60}$$

Estimation results for these alternative monetary policy rules are given in Table 4.4. We see that despite the differences in the level of abstraction, and despite the differences in the estimated parameter values, there are no substantial differences in terms of the model fit. As expected, the model fit measured by the value of the minimization problem $\Im$ is best for the estimated Taylor rule (TR), but it is nearly as good for strict domestic inflation targeting (DIT).

Table 4.4: Comparison of Results for Different Monetary Policy Rules

Criterion	TR	DIT	DITR	CITR	PEG
κ	0.39	0.42	0.48	0.90	0.41
θ	0.78	0.90	0.38	0.90	0.87
α	0.05	0.05	0.05	0.06	0.05
η	1.00	1.00	1.00	1.10	1.03
σ	3.15	3.11	1.67	1.14	1.62
Puzzle 1	19.36	19.37	19.13	19.05	19.06
Puzzle 2	-0.48	-0.65	-0.81	-0.75	0.46
Puzzle 3	0.97	0.97	0.97	0.99	0.97
Puzzle 4	2.83	2.85	1.59	1.65	1.56
Puzzle 51	1.97	1.96	1.47	1.11	1.44
Puzzle 52	0.61	0.61	0.61	0.61	0.61
Puzzle 61	3.13	3.09	1.88	1.27	1.84
Puzzle 62	0.63	0.63	0.56	0.56	0.56
Minimization $\Im$	36.10	36.43	43.44	51.33	43.93

4.5.3 Results for the GM Baseline Model

We have seen that a carefully estimated model with trade costs performs very well in the cross-validation of the puzzling data. But what about the original GM model? What if their "special case" calibration and their then optimal DIT policy is used? In that case, $\sigma = \eta = 1$, $\theta = 0.75$, $\alpha = 0.4$ and, of course, $\kappa = 0$. All other parameters are virtually the same as here. The result of this endeavor is presented in Table 4.5. For comparison reasons, I also add column three of Table 4.2, containing the moments of my TR parameter estimation. What we see from this is that the original GM model does very well compared to the estimated TR model. Its only comparative weakness is the very low exchange rate volatility.

Table 4.5: Comparison of the TR Model with the GM DIT Model

Criterion	Data	TR	GM DIT
Puzzle 1	$[1;19]$	19.36	1.50
Puzzle 2	$[-1;0]$	-0.48	-0.63
Puzzle 3	$[.32;.92]$	0.97	0.60
Puzzle 4	$[0.5;1]$	2.83	2.74
Puzzle 51	7.52	1.97	0.65
Puzzle 52	0.83	0.61	0.62
Puzzle 61	4.36	3.13	0.82
Puzzle 62	0.08	0.63	0.48

Notes: The GM DIT model is calibrated as suggested in GM, especially $\sigma = \eta = 1$, $\theta = 0.75$, $\alpha = 0.4$, and $\kappa = 0$. The TR model is parameterized as given in Table 4.1.

4.6 Conclusion

Can the Galí and Monacelli (2005b) model replicate the six major puzzles in international macroeconomics, as collected by Obstfeld and Rogoff (2000*b*)? At first glance, this seems to be a challenging endeavor: This model is highly stylized, with complete financial markets, no capital, and a minimum of shocks and frictions. Nonetheless, some insight might be obtained. This textbook model is widely used in academics and at central banks. It forms the way economists think about monetary policy in open economies. If the model deviates essentially from reality along the six puzzles, its usefulness should be doubted. So I have put up the fight between a stylized model and the rich and puzzling data. And it turns out to be a good one: Given the simplicity of the model, it performs quite well. This result holds true even for the case of a very stylized, close to optimal monetary policy in the small open economy.

Against expectance, the combination of two rather isomorphic ingredients – trade costs and a home bias in preferences – helps a lot to bring the model close to the data. So OR's assumption that trade costs do help in resolving the six puzzles proves true.

There are three big deficiencies for the model: First, the international correlation pattern of output and consumption, termed as quantity anomaly by Backus et al. (1995), is not met in any of the model specifications considered. All parameter combinations investigated result in a situation where international consumption correlation is higher than international output correlation. Given the simplicity of the model stochastics, this might simply be an artefact of the assumed productivity correlation. Indeed, changing the latter results in an improvement along this dimension. However, since this correlation is inherent in the data, its influence on the model accuracy will be neglected here. The second deficiency is the volatility of the real exchange rate, which still remains too low compared to the data. Nonetheless, compared to the original GM calibration, my baseline choice of parameter implies a strong increase in the real exchange rate volatility. For further increases, the literature has shown that pricing-to-market arrangements may help a lot, but this is left for future research. The third and last deficiency is the high correlation between real exchange rate and output, which is seen in all specifications of the model. As a remedy for this, one should again think about a richer set of stochastic elements in the model. Another promising topic is the inclusion of a more realistic fiscal policy instead of the production subsidy assumed so far.

If these deficiencies are important for a specific research question, one should not rely on the stylized New Keynesian small open economy model examined in this paper. Instead, one should look for a more elaborated model. In case these deficiencies are of minor importance, I have shown in Section 4.5.3 that even the textbook GM model is doing reasonably well against the six puzzles.

5 Technical Appendix

5.1 Technical Appendix to Chapter 2

5.1.1 First Order Conditions and Aggregate Demand

Consumption maximization is done in two steps: first, suppose that C_A^j is a single good instead of an aggregate. Then, maximization of the utility function (2.3) of agent j in region A subject to the corresponding aggregated budget constraint (2.9) implies the two first-order conditions

$$\lambda_{BC} = \left(\frac{C^j}{\gamma}\right)^{\gamma-1}\left(\frac{M_A^j/P^A}{1-\gamma}\right)^{1-\gamma}\nu\frac{C^j}{P_A^A C_A^j}, \tag{5.1}$$

$$\lambda_{BC} = \left(\frac{C^j}{\gamma}\right)^{\gamma}\left(\frac{M_A^j/P^A}{1-\gamma}\right)^{-\gamma}\frac{1}{P^A}. \tag{5.2}$$

Equalizing the two equations by replacing the Lagrange multiplier λ_{BC} and noting that $\frac{P^i C^j}{\gamma} = \frac{M_i^j}{1-\gamma} = I_i^j$ leads to[1]

$$C_A^j = \nu\left(\frac{P^A}{P_A^A}\right)C^j. \tag{5.3}$$

Second, maximizing C_A^j with respect to two generic elements $c^j(a)$ and $c^j(h')$, subject to $\int_0^n P^i(a)c^j(a)da = Z$, leads to

$$c^j(a) = \left(\frac{p^i(a)}{p^i(h')}\right)^{-\theta} c^j(h')\,. \tag{5.4}$$

[1] This is a result of the Cobb-Douglas structure of the utility function.

Then, replacing $c^j(a)$ in Equation (2.5) by the right-hand side of the previous equation gives

$$\begin{aligned} C_A^j &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n \left(\left(\frac{p^i(a)}{p^i(h')}\right)^{-\theta} c^j(h') \right)^{\frac{\theta-1}{\theta}} da \right]^{\frac{\theta}{\theta-1}} \\ &= p^i(h')^{\theta} c^j(h') n (P_A^i)^{-\theta}, \end{aligned}$$

which implies

$$c^j(a) = \frac{1}{n}\left(\frac{p^i(a)}{P_A^i}\right)^{-\theta} C_A^j. \tag{5.5}$$

Adding steps one and two plus the symmetric results for the foreign good – for ease of exposition agent j is still assumed to live in region A – results in[2]

$$c^j(a) = \frac{\nu}{n}\left(\frac{p^A(a)}{P_A^A}\right)^{-\theta} \frac{P^A}{P_A^A} C^j \quad \text{and} \quad c^j(b) = \frac{1-\nu}{1-n}\left(\frac{p^A(b)}{P_B^A}\right)^{-\theta} \frac{P^A}{P_B^A} C^j. \tag{5.6}$$

Using the terms of trade and the fact that $C^j = \frac{\gamma I_i^j}{P^i}$, we can rewrite the first-order condition of the producer-consumers with respect to their consumption of a single good and – in a similar manner – to their money holdings M_i^j as

$$c^j(a) = \frac{\nu}{n}\left(\frac{p^A(a)}{P_A}\right)^{-\theta} \frac{\gamma I_A^j}{P_A}, \tag{5.7}$$

$$c^j(b) = \frac{1-\nu}{1-n}\left(\frac{p^A(b)}{P_B}\right)^{-\theta} \frac{\gamma I_A^j}{P_B}, \tag{5.8}$$

$$c^j(a) = \frac{1-\nu}{n}\left(\frac{p^A(a)}{P_A}\right)^{-\theta} \frac{\gamma I_B^j}{P_A}, \tag{5.9}$$

$$c^j(b) = \frac{\nu}{1-n}\left(\frac{p^B(b)}{P_B}\right)^{-\theta} \frac{\gamma I_B^j}{P_B}, \tag{5.10}$$

$$M_i^j = (1-\gamma) I_i^j. \tag{5.11}$$

The first two equations determine a home resident's optimal choice of home and foreign goods, the next two equations determine the analog for a foreign resident, while the last equation shows the optimality condition with respect to money holdings.

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[2] An agent j of region B would demand $c^j(a) = \frac{1-\nu}{n}\left(\frac{p^A(a)}{P_A^B}\right)^{-\theta} \frac{P^B}{P_A^B} C^j$ and $c^j(b) = \frac{\nu}{1-n}\left(\frac{p^B(b)}{P_B^B}\right)^{-\theta} \frac{P^B}{P_B^B} C^j$.

Total nominal expenditure by consumers in region A is $I_A = \int_0^n I_A^j dj$, while in region B it is $I_B = \int_n^1 I_B^j dj$. The demand function for a good a is given by

$$\begin{aligned} Y^d(a) &= \int_0^1 c^j(a)dj \\ &= \left(\frac{p^A(a)}{P_A}\right)^{-\theta} \frac{1}{n}\left[\gamma \frac{\nu I_A + (1-\nu)I_B}{P_A}\right]. \end{aligned} \tag{5.12}$$

Similarly, the demand for a certain good b is given by

$$\begin{aligned} Y^d(b) &= \int_0^1 c^j(b)dj \\ &= \left(\frac{p^B(b)}{P_B}\right)^{-\theta} \frac{1}{1-n}\left[\gamma \frac{(1-\nu)I_A + \nu I_B}{P_B}\right]. \end{aligned} \tag{5.13}$$

Denoting “not i" by $-i$ and using the notation

$$w_i = \begin{cases} n & \text{if } i = A, \\ 1-n & \text{if } i = B \end{cases}$$

for the region sizes, we define a variable proportional to “wealth":

$$W \equiv \frac{\gamma}{w_i} \frac{\nu I_i + (1-\nu)I_{-i}}{P_i}. \tag{5.14}$$

At this point it is useful to note that this definition includes the terms of trade between domestic and foreign goods, as $I_i = \frac{P^i C}{\gamma}$ measures the nominal consumption expenditures using the level of the consumer price index (CPI), while the denominator involves the level of the producer price index (PPI) as a reference. Using the identities from (2.13), one can easily transform this notation into one that includes real expenditures and the terms of trade S:

$$W = \begin{cases} \nu(S_A)^{1-\nu}\left(\frac{\gamma}{n}\frac{I_A}{P^A}\right) + (1-\nu)(S_A)^{\nu}\left(\frac{\gamma}{1-n}\frac{I_B}{P^B}\right) & \text{if } i = A, \\ \nu(S_A)^{\nu-1}\left(\gamma\frac{I_B}{P^B}\right) + (1-\nu)(S_A)^{-\nu}\left(\gamma\frac{I_A}{P^A}\right) & \text{if } i = B. \end{cases}$$

Then, demand for a specific good j from region i amounts to

$$Y^d(j) = \left(\frac{p^i(j)}{P_i}\right)^{-\theta} W. \tag{5.15}$$

Analogously to Benigno (2004), the smaller a region is (i.e. the higher the degree of openness), the larger the terms of trade effect will be on regional output (included in the W term).[3]

5.1.2 Price Setting

When selling the product each producer is a monopolist. The producer, therefore, decides upon the price of the product by maximizing the indirect utility function. The indirect utility function is obtained by plugging $C^j = \frac{\gamma I_i^j}{P^i}$ and $M_i^j = (1-\gamma)I_i^j$ into the utility function (2.3), replacing I_i^j by the right-hand side of the budget constraint, replacing the price ratio with the help of Equation (5.15), and simplifying:

$$U_i^j = (1-\tau_i)W^{\frac{1}{\theta}}(Y_i^j)^{\frac{\theta-1}{\theta}} - T_i + \frac{\bar{M}_i^j}{P_i} - \left(\frac{\xi_i}{\beta}\right)(Y_i^j)^{\beta}. \tag{5.16}$$

The indirect utility function of agent j is maximized with respect to the price $p^i(j)$, noting that the output produced by agent j is equal to its demand, i.e. $Y_i^j = Y^d(j)$.[4]

We obtain the optimal ratio of prices as

$$\begin{aligned}\left(\frac{p^i(j)}{P_i}\right) &= \left(\frac{-\xi_i\theta W^{\beta-1}}{(1-\tau_i)(1-\theta)}\right)^{\frac{1}{-\theta+\theta\beta+1}} \\ &= \left(\frac{\theta\xi_i}{(\theta-1)(1-\tau_i)}W^{\beta-1}\right)^{\frac{1}{1+\theta(\beta-1)}} .\end{aligned} \tag{5.17}$$

Writing the last equation for each region separately, we have

$$\left(\frac{p(a)}{P_A}\right) = \left(\frac{\theta\xi_A}{(\theta-1)(1-\tau_A)}W^{\beta-1}\right)^{\frac{1}{1+\theta(\beta-1)}} \quad \text{and} \quad \left(\frac{p(b)}{P_B}\right) = \left(\frac{\theta\xi_B}{(\theta-1)(1-\tau_B)}W^{\beta-1}\right)^{\frac{1}{1+\theta(\beta-1)}}. \tag{5.18}$$

Furthermore, we assume that some prices are fixed in advance, comparable to a static version of the staggered price-setting introduced by Calvo (1983). A fraction Φ^i of producers cannot change their prices and thus have to charge the same prices as in the past, whereas a fraction $(1-\Phi^i)$ of producers are able to set their prices freely after the realization of the shocks in region i. The price level of goods from region A is a

[3]Note that our demand functions are more complicated than the ones in Benigno because of the preference parameter ν. This destroys the identity $P^A = P^B$ that holds in Benigno (2004) as long as $\nu^A \neq n$. If $\nu^A = \nu^B = n$ and $1-\nu^A = 1-\nu^B = 1-n$, the consumer price indices of both regions are identical, and the demand functions become as simple as in Benigno.

[4]As the decision of a single individual has only marginal impact on terms of trade and the price indices, this effect is neglected in the optimization.

weighted sum of the average of pre-set prices $E[\bar{p}^A(a)]$ and the newly set prices $\tilde{p}^A(a)$, which due to symmetry are equal for all producers. Based on Equation (2.7), we obtain

$$P_A^{1-\theta} = \Phi^A(E\bar{p}^A(a))^{1-\theta} + (1-\Phi^A)(\tilde{p}^A(a))^{1-\theta}. \tag{5.19}$$

For goods produced in region B the equivalent equation is

$$P_B^{1-\theta} = \Phi^B(E\bar{p}^B(b))^{1-\theta} + (1-\Phi^B)(\tilde{p}^B(b))^{1-\theta}. \tag{5.20}$$

For convenience, the price ratio in region i may be defined to be

$$\lambda_i \equiv \Phi^i\left(\frac{E\bar{p}^i(j)}{P_i}\right)^{1-\theta} + (1-\Phi^i)\left(\frac{\tilde{p}^i(j)}{P_i}\right)^{1-\theta} = 1. \tag{5.21}$$

In line with Equation (2.6) the aggregate consumer price index in region i is given by

$$\begin{aligned} P^A = &\left[\Phi^A\left(E\bar{p}^A(a)\right)^{1-\theta} + (1-\Phi^A)\left(\tilde{p}^A(a)\right)^{1-\theta}\right]^{\frac{\nu}{1-\theta}} \\ &\times\left[\Phi^B\left(E\bar{p}^B(b)\right)^{1-\theta} + (1-\Phi^B)\left(\tilde{p}^B(b)\right)^{1-\theta}\right]^{\frac{1-\nu}{1-\theta}} \end{aligned} \tag{5.22}$$

$$\begin{aligned} P^B = &\left[\Phi^B\left(E\bar{p}^B(b)\right)^{1-\theta} + (1-\Phi^B)\left(\tilde{p}^B(b)\right)^{1-\theta}\right]^{\frac{\nu}{1-\theta}} \\ &\times\left[\Phi^A\left(E\bar{p}^A(a)\right)^{1-\theta} + (1-\Phi^A)\left(\tilde{p}^A(a)\right)^{1-\theta}\right]^{\frac{1-\nu}{1-\theta}}. \end{aligned} \tag{5.23}$$

This can be written in terms of the overall price level[5]

$$\begin{aligned} P \equiv &(P^A)^n(P^B)^{1-n} \\ = &\left[\Phi^A\left(E\bar{p}^A(a)\right)^{1-\theta} + (1-\Phi^A)\left(\tilde{p}^A(a)\right)^{1-\theta}\right]^{\frac{n\nu+(1-n)(1-\nu)}{1-\theta}} \\ &\times\left[\Phi^B\left(E\bar{p}^B(b)\right)^{1-\theta} + (1-\Phi^B)\left(\tilde{p}^B(b)\right)^{1-\theta}\right]^{\frac{n(1-\nu)+(1-n)\nu}{1-\theta}}. \end{aligned} \tag{5.24}$$

5.1.3 Aggregate Output and Fiscal Policy

Aggregate output in each region is defined by the following equations:

$$Y_A \equiv \int_0^n \frac{p^A(a)Y(a)}{P_A}da \quad \text{and} \quad Y_B \equiv \int_n^1 \frac{p^B(b)Y(b)}{P_B}db. \tag{5.25}$$

[5] Note that the numerators of the exponents add up exactly to one.

Using the demand functions (5.12) and (5.13) as well as the price index definitions (2.7), and denoting the lower and upper integral limits of each region i by lli and uli, respectively,[6] aggregate output produced in region i can be rewritten as

$$\begin{aligned} Y_i &= \int_{lli}^{uli} \frac{p^i(j)}{P_i}\left(\frac{p^i(j)}{P_i}\right)^{-\theta} W dj = W\int_{lli}^{uli}\left(\frac{p^i(j)}{P_i}\right)^{1-\theta} dj \\ &= w_i\lambda_i W = w_i W. \end{aligned} \tag{5.26}$$

Essentially, this implies that the goods' supply in region i is equal to its demand, which according to Equation (5.14) originates from both regions. Total output is given as the geometric average of output in both regions:

$$Y \equiv Y_A^n Y_B^{1-n}. \tag{5.27}$$

As fiscal policy uses per-capita taxes T_i to subsidize production, i. e., $T_i > 0, \tau_i < 0$. wealth W simplifies to

$$\begin{aligned} W &= \frac{\gamma}{w_i}\frac{\nu I_i + (1-\nu)I_{-i}}{P_i} \\ &= \frac{\gamma}{w_i P_i}\left[\nu\int_{lli}^{uli} I_i^j dj + (1-\nu)\int_{ll-i}^{ul-i} I_{-i}^j dj\right] \\ &= \frac{\gamma}{w_i P_i}[\nu\int_{lli}^{uli}\left(p^i(j)Y_i^j(1-\tau_i) - P_i T_i + \bar{M}_i^j\right)dj \\ &+ (1-\nu)\int_{ll-i}^{ul-i}\left(p^{-i}(j)Y_{-i}^j(1-\tau_{-i}) - P_{-i}T_{-i} + \bar{M}_{-i}^j\right)dj] \\ &= \frac{\gamma}{w_i P_i}\left[\nu\left(P_i Y_i(1-\tau_i) - P_i T_i + \bar{M}_i\right) + (1-\nu)\left(P_{-i}Y_{-i}(1-\tau_{-i}) - P_{-i}T_{-i} + \bar{M}_{-i}\right)\right] \\ &= \frac{\gamma}{w_i P_i}\left[\nu\bar{M}_i + (1-\nu)\bar{M}_{-i} + \nu P_i w_i W + (1-\nu)P_{-i}w_{-i}W\right] \\ \Leftrightarrow W &= \frac{\gamma}{w_i}\frac{\bar{M}}{P}\frac{1}{1-\gamma[\nu + (1-\nu)\frac{w_{-i}}{w_i}S_i]}, \end{aligned} \tag{5.28}$$

where we assume identical beginning-of-period real money holdings for all agents $\frac{\bar{M}}{P} = \frac{\bar{M}_i^j}{P_i}$ and for all $i, -i$.[7]

Combining Equations (5.26) and (5.28), we can relate regional output to real money

[6] I.e., $lli = \begin{cases} 0 & \text{if } i = A, \\ n & \text{if } i = B, \end{cases}$ and $uli = \begin{cases} n & \text{if } i = A, \\ 1 & \text{if } i = B. \end{cases}$

[7] Without the assumption of internationally identical money holdings $\bar{M}/P$ has to be replaced by $[n\bar{M}_i + (1-n)\bar{M}_{-i}]/P_i$.

balances as follows:

$$Y_A = \gamma \frac{\bar{M}}{P} \frac{\gamma}{1-\gamma[\nu + (1-\nu)\frac{1-n}{n} S_A]} \qquad \text{and} \qquad Y_B = \gamma \frac{\bar{M}}{P} \frac{1}{1-\gamma[\nu + (1-\nu)\frac{n}{1-n} S_B]}, \quad (5.29)$$

which are the equations given in Section 2.2.2 in the main text.

5.1.4 Log-Linear Equilibrium Fluctuations: Price Setting

We log-linearize the model as follows: First, note that a linear approximation of Equation (2.6) around $P_i = P^i = P$ for all i results in

$$\pi^A = \nu\pi_A + (1-\nu)\pi_B \qquad \text{and} \qquad \pi^B = \nu\pi_B + (1-\nu)\pi_A, \quad (5.30)$$

where the inflation rates are defined as percentage deviations of the respective price level from its steady-state level,[8] i.e.

$$\pi^i \equiv \log(P^i) - \log(\bar{P}^i), \text{ given } \bar{P}^i \neq 0. \quad (5.31)$$

Then, Equations (5.19) and (5.20) linearize[9] to

$$\pi_A = \Phi^A \bar{\pi}_A + (1-\Phi^A)\tilde{\pi}_A \qquad \text{and} \qquad \pi_B = \Phi^B \bar{\pi}_B + (1-\Phi^B)\tilde{\pi}_B. \quad (5.32)$$

Combining the results gives

$$\pi^A = \nu(\Phi^A \bar{\pi}_A + (1-\Phi^A)\tilde{\pi}_A) + (1-\nu)(\Phi^B \bar{\pi}_B + (1-\Phi^B)\tilde{\pi}_B) \quad (5.33)$$

$$\pi^B = \nu(\Phi^B \bar{\pi}_B + (1-\Phi^B)\tilde{\pi}_B) + (1-\nu)(\Phi^A \bar{\pi}_A + (1-\Phi^A)\tilde{\pi}_A). \quad (5.34)$$

Now, we turn to the optimal price a producer would set if he could choose the price freely. According to Dixit and Lambertini (2003*a*), we refer to the idea of Calvo staggered pricing, which reflects a dynamic setting (for details see Calvo 1983). Analogously to the procedure proposed by Dixit and Lambertini, we introduce a discount factor η with $\eta < 1$ (which means that pseudo-future period utilities have a lower weight than present utility). We, first, assume that η equals unity to explain the "in-

[8] Under the assumption that $\bar{P}^i \equiv 1$, one can equivalently define $\pi^i \equiv \log(P^i)$.

[9] To appreciate this, compare the following procedure undertaken with a simplified, yet similar equation:
$P^b = \phi Q^b + (1-\phi)R^b \quad \Rightarrow \quad \bar{P}^b e^{b\pi} = \phi \bar{Q}^b e^{b\bar{\pi}} + (1-\phi)\bar{R}^b e^{b\tilde{\pi}}$, which is approximately equal to
$\bar{P}^b(1+b\pi) = \phi\bar{Q}^b(1+b\bar{\pi}) + (1-\phi)\bar{R}^b(1+b\tilde{\pi}) \quad \Rightarrow \quad b\pi = \phi\frac{\bar{Q}^b}{\bar{P}^b} b\bar{\pi} + (1-\phi)\frac{\bar{R}^b}{\bar{P}^b} b\tilde{\pi}$. As the fractions are equal to unity, this simplifies to $\pi = \phi\bar{\pi} + (1-\phi)\tilde{\pi}$.

tuitional proceeding". In the case where prices are allowed to change, the optimal log price equals

$$\tilde{\pi}_A = (1-\Phi^A)\pi_A^j + \Phi^A\bar{\pi}_A$$
$$\tilde{\pi}_B = (1-\Phi^B)\pi_B^j + \Phi^B\bar{\pi}_B.$$

where π_i^j is the log steady-state deviation of the price that would be optimal if prices could be adjusted freely. The log price set by producer j is a sum of the weighted optimal price of producer j, if prices were fully flexible, and the weighted price that maximizes the expected indirect utility, if prices are to be fixed in future periods. The weights equal the probability of being able, $(1-\Phi^i)$, or not being able, Φ^i, to change the price in the following period(s).

Now we return to the discount factor $\eta < 1$. As already mentioned, the individuals place lower weight on future utilities. Therefore, the fact that the producer cannot change the price in future periods with a certain probability is expressed by a lower weight than the pure probability of future price setting (given by $\eta\Phi^i$) and a higher weight for the present period $(1-\eta\Phi^i)$. Hence, we obtain

$$\tilde{\pi}_A = (1-\Phi^A\eta)\pi_A^j + \Phi^A\eta\bar{\pi}_A, \tag{5.35}$$
$$\tilde{\pi}_B = (1-\Phi^B\eta)\pi_B^j + \Phi^B\eta\bar{\pi}_B. \tag{5.36}$$

In the case of $\eta = 0$, this setting would be purely static: Here, the (deviation from the steady state of the) optimal price once an individual is allowed to change price $\tilde{\pi}_i$ is identical to the price that is optimal for the current period only, as there are no future periods to form expectations about.

Using Equations (5.35) and (5.36) to replace the optimal prices in the consumer price indices (5.33) and (5.34) gives

$$\begin{aligned}\pi^A &= \nu\Phi^A[1+(1-\Phi^A)\eta]\bar{\pi}_A + \nu(1-\Phi^A)(1-\Phi^A\eta)\pi_A^j \\ &\quad + (1-\nu)\Phi^B[1+(1-\Phi^B)\eta]\bar{\pi}_B + (1-\nu)(1-\Phi^B)(1-\Phi^B\eta)\pi_B^j \end{aligned} \tag{5.37}$$
$$\begin{aligned}\pi^B &= \nu\Phi^B[1+(1-\Phi^B)\eta]\bar{\pi}_B + \nu(1-\Phi^B)(1-\Phi^B\eta)\pi_B^j \\ &\quad + (1-\nu)\Phi^B[1+(1-\Phi^B)\eta]\bar{\pi}_A + (1-\nu)(1-\Phi^B)(1-\Phi^B\eta)\pi_A^j\,. \end{aligned} \tag{5.38}$$

The overall inflation rate can be calculated by using the previous equations together with Equation (5.24):

$$\pi = n\pi^A + (1-n)\pi^B \tag{5.39}$$

$$= [n\nu + (1-n)(1-\nu)]\pi_A + [n(1-\nu) + (1-n)\nu]\pi_B. \tag{5.40}$$

Equation (5.39) states that union-wide inflation is the sum of the regional CPI inflation weighted by the size of each region. The second Equation (5.40) links union-wide inflation to the PPI inflation rates in each region, where the influence of regional PPI inflation depends on both the size of the region and the preference of agents for goods from that region.

5.1.5 Proof of Proposition 1: Inflation Determination

In general, a producer sets its price by maximizing the indirect utility function which results in Equation (5.17) above. A log-linear approximation of this equation around the steady state, solved for the relative deviation of wealth from its steady state level, $\hat{W}$, is

$$\hat{W} = \frac{1+\theta(\beta-1)}{\beta-1}(\hat{p}^i(j) - \pi_i) - \frac{1}{\beta-1}\hat{\xi}_i - \frac{\bar{\tau}_i}{\beta-1}\hat{\tau}_i, \tag{5.41}$$

where $\pi_i \equiv \hat{P}_i$, and a "hat" above a variable denotes percentage deviations of the variable from its steady state.[10] To replace $\hat{W}$ in the last expression, we log-linearize the policy dependent wealth equation.

For the fiscal policy considered here, we use Equation (5.28), and obtain the result

$$\hat{W} = \frac{\gamma\bar{m}}{\omega_i w_i}\hat{m} + \frac{\gamma(1-\nu)}{\omega_i}\frac{w_{-i}}{w_i}s_i, \tag{5.42}$$

where ω_i is given by $\omega_i \equiv 1 - \gamma[\nu + (1-\nu)\frac{w_{-i}}{w_i}]$ and $s_i \equiv \hat{S}_i = \pi_{-i} - \pi_i$. $\hat{m}$ is the change in the beginning-of-period real money holdings $\bar{M}/P$.

In the next step, Equation (5.42) is evaluated at both $E[\hat{\bar{p}}^i(j)] \equiv \bar{\pi}_i$, the (log deviation of the) price that maximizes the future indirect utility, and at $\hat{p}_i^j \equiv \pi_i^j$, the (log deviation of the) price that maximizes the current period indirect utility. Starting with the first

[10] For the approximation of the fiscal policy term, note that $\widehat{(1-\tau_i)} = \frac{-\bar{\tau}_i}{1-\bar{\tau}_i}\hat{\tau}_i$.

case $\bar{\pi}_i$, we obtain

$$
\begin{aligned}
\bar{\pi}_i &= E[\pi_i] + \frac{1}{1+\theta(\beta-1)} E[\hat{\xi}_i] + \frac{\bar{\tau}_i}{1+\theta(\beta-1)} E[\hat{\tau}_i] \\
&\quad + \frac{\beta-1}{1+\theta(\beta-1)} E\left[\frac{\gamma \bar{m}}{\omega_i w_i} \hat{m} + \frac{\gamma(1-\nu)}{\omega_i} \frac{w_{-i}}{w_i} s_i\right] \\
&= \bar{\omega}_{0,i} + \omega_1 E[\hat{\tau}_i] + \omega_2 E[\hat{\tau}_{-i}] + \omega_3 E[\pi_i] + \omega_4 E[\pi_{-i}], \qquad (5.43)
\end{aligned}
$$

where $\bar{\omega}_{0,i} \equiv \frac{1}{1+\theta(\beta-1)} E[\hat{\xi}_i] + \frac{\beta-1}{1+\theta(\beta-1)} \frac{\gamma \bar{m}}{\omega_i w_i} E[\hat{m}]$, $\omega_1 \equiv \frac{\bar{\tau}_i}{1+\theta(\beta-1)}$, $\omega_2 \equiv 0$, $\omega_3 \equiv \left(1 - \frac{\beta-1}{1+\theta(\beta-1)} \frac{\gamma(1-\nu)}{\omega_i} \frac{w_{-i}}{w_i}\right)$ and $\omega_4 \equiv \frac{\beta-1}{1+\theta(\beta-1)} \frac{\gamma(1-\nu)}{\omega_i} \frac{w_{-i}}{w_i}$.[11] Note that s_i has been replaced by terms of π_i and π_{-i}. Accordingly, for the price that maximizes the current period indirect utility only, we obtain

$$
\begin{aligned}
\pi_i^j &= \frac{1}{1+\theta(\beta-1)} \hat{\xi}_i + \frac{\beta-1}{1+\theta(\beta-1)} \frac{\gamma \bar{m}}{\omega_i w_i} \hat{m} + \frac{\bar{\tau}_i}{1+\theta(\beta-1)} \hat{\tau}_i \\
&\quad + \left(1 - \frac{\beta-1}{1+\theta(\beta-1)} \frac{\gamma(1-\nu)}{\omega_i} \frac{w_{-i}}{w_i}\right) \pi_i + \frac{\beta-1}{1+\theta(\beta-1)} \frac{\gamma(1-\nu)}{\omega_i} \frac{w_{-i}}{w_i} \pi_{-i} \qquad (5.44) \\
&= \omega_{0,i} + \frac{\beta-1}{1+\theta(\beta-1)} \frac{\gamma \bar{m}}{\omega_i w_i} \hat{m} + \omega_1 \hat{\tau}_i + \omega_2 \hat{\tau}_{-i} + \omega_3 \pi_i + \omega_4 \pi_{-i}, \qquad (5.45)
\end{aligned}
$$

where $\omega_{0,i} = \frac{1}{1+\theta(\beta-1)} \hat{\xi}_i$ captures the stochastic term in this equation. Using Equations (5.32), (5.35) and (5.36), we obtain an equation that expresses the regional producer inflation rate in terms of the log of the price that maximizes the future indirect utility and the prize that maximizes the current period indirect utility only:

$$
\pi_i = \rho^i \bar{\pi}_i + (1-\rho^i) \pi_i^j, \qquad \rho^i = \Phi^i [1 + (1-\Phi^i)\eta]. \qquad (5.46)
$$

The parameter ρ^i bears a similar intuition as the "Calvo" parameter Φ^i: a value of zero corresponds to completely flexible prices, whereas a value of unity implies that all prices cannot change in response to shocks.

We use (5.46) and combine the two log prices in equations (5.43) and (5.45):

$$
\begin{aligned}
\pi_i &= \rho^i \left[\bar{\omega}_{0,i} + \omega_1 E[\hat{\tau}_i] + \omega_2 E[\hat{\tau}_{-i}] + \omega_3 E[\pi_i] + \omega_4 E[\pi_{-i}]\right] \\
&\quad + (1-\rho^i) \left[\omega_{0,i} + \frac{\beta-1}{1+\theta(\beta-1)} \frac{\gamma \bar{m}}{\omega_i w_i} \hat{m} + \omega_1 \hat{\tau}_i + \omega_2 \hat{\tau}_{-i} + \omega_3 \pi_i + \omega_4 \pi_{-i}\right]. \qquad (5.47)
\end{aligned}
$$

[11] We add the term ω_2 to show that under alternative fiscal policies this spillover effect can be non-zero.

For the other region, analog steps yield

$$\pi_{-i} = \rho^i \left[\bar{\omega}_{0,-i} + \omega_1 E[\hat{\tau}_{-i}] + \omega_2 E[\hat{\tau}_i] + \omega_3 E[\pi_{-i}] + \omega_4 E[\pi_i]\right] + (1-\rho^i)\left[\omega_{0,-i} + \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma\bar{m}}{\omega_{-i}w_{-i}}\hat{m} + \omega_1\hat{\tau}_{-i} + \omega_2\hat{\tau}_i + \omega_3\pi_{-i} + \omega_4\pi_i\right], \quad (5.48)$$

where $\omega_{0,-i}$ differs only from $\omega_{0,i}$ by the respective regional stochastic productivity variable $\hat{\xi}_{-i}$ instead of $\hat{\xi}_i$.

Combining (5.47) and (5.48) and solving this system of equations for the region-specific inflation rates, one obtains

$$\begin{aligned}
\pi_i = {} & \Omega\rho^i \left[\bar{\omega}_{0,i} + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\bar{\omega}_{0,-i}\right] \\
& + \Omega\rho^i \left[\left(\bar{\omega}_1 + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\omega_2\right)E[\hat{\tau}_i] + \left(\omega_2 + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\omega_1\right)E[\hat{\tau}_{-i}]\right] \\
& + \Omega\rho^i \left[\left(\omega_3 + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\omega_4\right)E[\pi_i] + \left(\omega_4 + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\omega_3\right)E[\pi_{-i}]\right] \\
& + \Omega(1-\rho^i)\left[\omega_{0,i} + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\omega_{0,-i}\right] \\
& + \Omega(1-\rho^i)\left(\frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma\bar{m}}{\omega_i w_i} + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma\bar{m}}{\omega_{-i}w_{-i}}\right)\hat{m} \\
& + \Omega(1-\rho^i)\left(\omega_1 + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\omega_2\right)\hat{\tau}_i \\
& + \Omega(1-\rho^i)\left(\omega_2 + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\omega_1\right)\hat{\tau}_{-i}
\end{aligned} \quad (5.49)$$

with $\Omega \equiv \frac{1-(1-\rho^i)\omega_3}{[1-(1-\rho^i)\omega_3]^2-[(1-\rho^i)\omega_4]^2}$. Under the supply-side fiscal policy introduced above,[12] we have $\omega_2 = 0$. We can rewrite the last equation in terms of policy variables as follows:

$$\pi_i = d^i\hat{m} + c^i\hat{\tau}_i + c^{-i}\hat{\tau}_{-i} + \psi_i, \quad i \in \{A, B\}, \quad (5.50)$$

where the parameters are given by

$$c^i \equiv \Omega(1-\rho^i)\omega_1, \quad (5.51)$$

$$c^{-i} \equiv \Omega(1-\rho^i)\frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\omega_1 \quad \text{and} \quad (5.52)$$

$$d^i \equiv \Omega(1-\rho^i)\frac{\gamma\bar{m}(\beta-1)}{1+\theta(\beta-1)}\left(\frac{1}{\omega_i w_i} + \frac{1}{\omega_{-i}w_{-i}}\frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\right), \quad (5.53)$$

[12] Our setting could be easily enhanced for other types of fiscal policies.

and the variable ψ_i captures the expectational and stochastic terms,

$$\begin{aligned}\psi_i \equiv \Omega\rho^i &\left[\bar{\omega}_{0,i} + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\bar{\omega}_{0,-i}\right] + \Omega\rho^i \left[\omega_1 E[\hat{\tau}_i] + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\omega_1 E[\hat{\tau}_{-i}]\right] \\ &+ \Omega\rho^i \left[\left(\omega_3 + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\omega_4\right) E[\pi_i] + \left(\omega_4 + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\omega_3\right) E[\pi_{-i}]\right] \\ &+ \Omega(1-\rho^i)\left[\omega_{0,i} + \frac{(1-\rho^i)\omega_4}{1-(1-\rho^i)\omega_3}\omega_{0,-i}\right] \qquad \blacksquare\end{aligned}$$

5.1.6 Proof of Proposition 2: Output Determination

To obtain the equation for regional output y_i, we start with Equation (5.26) and plug in Equation (5.17):

$$\begin{aligned}Y_i &= W\int_{lli}^{uli}\left(\frac{p^i(j)}{P_i}\right)^{1-\theta} dj \\ &= \left(\frac{\theta\xi_i}{(\theta-1)(1-\tau_i)}W^{\beta-1}\right)^{\frac{1-\theta}{1+\theta(\beta-1)}} W.\end{aligned}$$

Log-linearizing this equation and using the notation $y_i \equiv \hat{Y}_i$, we get

$$y_i = \frac{1-\theta}{1+\theta(\beta-1)}\hat{\xi}_i + \frac{1-\theta}{1+\theta(\beta-1)}\bar{\tau}_i\hat{\tau}_i + \frac{(\beta-1)(1-\theta)}{1+\theta(\beta-1)}\hat{W} + \hat{W}. \tag{5.54}$$

Now we follow the procedure in Dixit and Lambertini (2003*b*) and apply Equation (5.41) in two ways. First, we replace the first $\hat{W}$ in (5.41) with i indices and the second $\hat{W}$ with $-i$ indices. We thus obtain

$$\begin{aligned}y_i = &\frac{1-\theta}{1+\theta(\beta-1)}\hat{\xi}_i + \frac{1-\theta}{1+\theta(\beta-1)}\bar{\tau}_i\hat{\tau}_i \\ &+ \frac{(\beta-1)(1-\theta)}{1+\theta(\beta-1)}\left[\frac{1+\theta(\beta-1)}{\beta-1}(\hat{p}^i(j)-\pi_i) - \frac{1}{\beta-1}\hat{\xi}_i - \frac{\bar{\tau}_i}{\beta-1}\hat{\tau}_i\right] \\ &+ \left[\frac{1+\theta(\beta-1)}{\beta-1}(\hat{p}^{-i}(j)-\pi_{-i}) - \frac{1}{\beta-1}\hat{\xi}_{-i} - \frac{\bar{\tau}_{-i}}{\beta-1}\hat{\tau}_{-i}\right].\end{aligned} \tag{5.55}$$

Second, we do the same thing the other way round, leading to

$$
\begin{aligned}
y_i = {} & \frac{1-\theta}{1+\theta(\beta-1)}\hat{\xi}_i + \frac{1-\theta}{1+\theta(\beta-1)}\bar{\tau}_i\hat{\tau}_i \\
& + \frac{(\beta-1)(1-\theta)}{1+\theta(\beta-1)}\left[\frac{1+\theta(\beta-1)}{\beta-1}(\hat{p}^{-i}(j)-\pi_{-i}) - \frac{1}{\beta-1}\hat{\xi}_{-i} - \frac{\bar{\tau}_{-i}}{\beta-1}\hat{\tau}_{-i}\right] \\
& + \left[\frac{1+\theta(\beta-1)}{\beta-1}(\hat{p}^{i}(j)-\pi_{i}) - \frac{1}{\beta-1}\hat{\xi}_{i} - \frac{\bar{\tau}_{i}}{\beta-1}\hat{\tau}_{i}\right].
\end{aligned} \tag{5.56}
$$

In the next step, we add up the two equations and divide by two. We evaluate $\hat{p}^i(j)$ in both regions for the flexible price firms, i. e. we replace $\hat{p}^i(j)$ by π_i^j, the price that maximizes current period indirect utility only. Replacing π_i^j with Equation (5.46) and simplifying leads to

$$
\begin{aligned}
y_i = {} & \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\bar{\tau}_i\hat{\tau}_i - \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)\bar{\tau}_{-i}\hat{\tau}_{-i} \\
& + \frac{2\beta\rho^i}{(\beta-1)(1-\rho^i)}(\pi_i - \bar{\pi}_i) + \frac{\beta\rho^i}{(\beta-1)(1-\rho^i)}(s_i - \bar{s}_i) \\
& + \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\hat{\xi}_i - \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)\hat{\xi}_{-i}.
\end{aligned} \tag{5.57}
$$

The notation $\bar{s}_i = E[s_i]$ is used to denote region i's expected terms of trade. Given the steady state of $\bar{P}_i = \bar{P}^i = \bar{P}$ for all i, we have $\bar{s}_i \equiv 0$ so that we can drop this term. For ease of exposition, we rewrite the last equation as follows:

$$
y_i = a^i\hat{\tau}_i + a^{i,-i}\hat{\tau}_{-i} + b^i(\pi_i - \pi_i^e) + \kappa^i s_i + \phi_i, \tag{5.58}
$$

where $a^i \equiv \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\bar{\tau}_i$, $a^{i,-i} \equiv -\left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)\bar{\tau}_{-i}$,
$b^i \equiv \frac{2\beta\rho^i}{(\beta-1)(1-\rho^i)}$, with $\pi_i^e = \bar{\pi}_i = E[\pi_i]$, $\kappa^i \equiv \frac{\beta\rho^i}{(\beta-1)(1-\rho^i)}$ and

$$
\phi_i = \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\hat{\xi}_i - \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)\hat{\xi}_{-i}. \qquad \blacksquare
$$

5.1.7 Welfare Criterion for Fiscal Policy

In this subsection, we derive the fiscal welfare criterion. The fiscal authority of region i is assumed to maximize the utility of a representative agent in her specific region.

Steady-State Considerations

According to Equation (2.3) the representative agent j living in region i derives utility from consumption, real money balances, and leisure.

Assuming flexible prices in the steady state and neglecting the utility stemming from real balances, we can rewrite the indirect utility function (5.16) as

$$U_i^j = (1-\tau_i)Y_i^j - T_i - \frac{\xi_i}{\beta}(Y_i^j)^\beta, \tag{5.59}$$

where the fiscal variables T_i (lump-sum tax or transfer) and τ_i (tax or subsidy on the consumption goods) are inserted.

We consider a very simple fiscal policy, which counteracts frictions from monopolistic competition and the regional productivity shock. Furthermore, we assume a balanced budget which implies the following resource restricton:

$$T_i = -Y_i\tau_i\ . \tag{5.60}$$

Assuming a steady state in which all stochastic terms are equal to their expected value, i. e. $\bar{\xi}_i = E[\xi_i]$, the steady state *natural* level of output in the flexible price scenario equals

$$\bar{Y}_i^{j,n} = \left(\frac{(\theta-1)(1-\bar{\tau}_i)}{\theta\bar{\xi}_i}\right)^{\frac{1}{\beta-1}}. \tag{5.61}$$

We will approximate utility around this steady state given in equation (5.61), i. e. we linearize the utility function around the flex-price scenario, where at the same time stochastic terms are equal to their expected values. Note, however, that we do not necessarily assume that fiscal policy is efficient. Thus, the overall distortion in the steady state output level is a result of both, market power and fiscal policy.[13] If taxes were set efficiently, they would eliminate the distortions stemming from monopolistic power. Using Equation (5.61) and the derivative of Equation (5.59) with respect to τ_i yields the efficient tax rate

$$\tau_i^{\text{eff}} = \frac{1}{1-\theta}\ , \quad \theta > 1, \tag{5.62}$$

which can be inserted into Equation (5.61) to calculate the corresponding level of output in region i. Then,

$$Y_i^{j,\text{eff}} = \left(\frac{\theta-1}{\theta\xi_i - 1}\right)^{\frac{1}{\beta-1}}. \tag{5.63}$$

[13]See Woodford (2003, pp. 293f). His parameter Φ_y equals our parameter κ used later in this section, where we keep with the notation of Dixit and Lambertini (2003*a*), Appendix B.

Second-order Approximation of the Utility Function

A welfare maximizing fiscal policy in the home region optimizes the utility function of a representative agent j living in region A, which is given by Equation (2.3). We aggregate all agents living in region A, which simplifies the consumption part due to the symmetry of this problem, while it leaves us with an integral in the disutility-term, as staggered pricing implies a different behavior for different agents.

We obtain

$$U^A = \gamma \left(\frac{C^A}{\gamma}\right)^{\gamma} \left(\frac{M_A/P^A}{1-\gamma}\right)^{1-\gamma} - \frac{1}{n}\int_0^n \left(\frac{\xi_A}{\beta}\right)(Y_A^j)^{\beta} dj. \tag{5.64}$$

Note that we do not consider the fraction of utility that originates from real balances, as we focus on the cashless limit, following the seminal work of Rotemberg and Woodford (1998). Therefore, we only consider the fraction γ stemming from the $u(.)$-term.

The notation ξ is used to capture all stochastics of the model. [14] We approximate around the flexible price steady state level of consumption of households in region A, which is denoted by $\bar{C}$, henceforth.

Approximation of the Consumption Utility Part

We begin with the approximation of the $u(.)$-part in the utility function (5.64) around its steady state level under flexible prices and a given, constant fiscal policy by using a second-order Taylor series:

$$\begin{aligned}\tilde{u} = \gamma \Big(& \bar{u} + u_C \tilde{C} + u_\xi \tilde{\xi}_A + u_m \tilde{m} + \frac{1}{2} u_{CC} \tilde{C}^2 + \frac{1}{2} u_{\xi\xi} \tilde{\xi}_A^2 + \frac{1}{2} u_{mm} \tilde{m}^2 + \\ & u_{Cm} \tilde{C}\tilde{m} + u_{C\xi} \tilde{C}\tilde{\xi}_A + u_{m\xi} \tilde{m}\tilde{\xi}_A \Big) + \mathcal{O}(||\xi_A||^3) \,,\end{aligned} \tag{5.65}$$

where a variable with a tilde (e.g. $\tilde{X}$) denotes the absolute deviation from the respective steady state level ($\bar{X}$), i. e. for home consumption we define $\tilde{C} \equiv C - \bar{C}$. The term $\mathcal{O}(||\xi_A||^3)$ summarizes terms of third or higher order, and are thus neglected.

A subscript on u or v denotes the first derivative of u or v with respect to the argument indicated by the subscript (u_C is for example the first derivative of u with respect to consumption C). Correspondingly, we use two subscripts after u or v to denote second

[14] Placing the regional productivity shock ξ_A after the semicolon in the disutility-part of Equation (5.64) indicates direct dependency of the realization of the productivity shock.

derivatives. Furthermore, we use the notation m for domestic real money balance, i. e. $m \equiv M_A/P^A$. Then, we can rewrite Equation (5.65) as

$$\tilde{u} = \gamma\left(\bar{u} + u_C\tilde{C} + u_m\tilde{m} + \frac{1}{2}u_{CC}\tilde{C}^2 + \frac{1}{2}u_{mm}\tilde{m}^2 + u_{Cm}\tilde{C}\tilde{m} + u_{C\xi}\tilde{C}\tilde{\xi}_A\right)$$
$$+ \text{t.i.p.} + \mathcal{O}(||\xi_A||^3) \;, \tag{5.66}$$

where t.i.p. summarizes the terms that are independent of policy.

Combining the first order conditions of the utility function with respect to consumption and money holdings leads to

$$M_A = (1-\gamma)I_A = \frac{1-\gamma}{\gamma}P^A C \Leftrightarrow m = \frac{1-\gamma}{\gamma}C \,. \tag{5.67}$$

After log-linearization, we apply this equation for $\tilde{m}$ with $\tilde{m} = \frac{1-\gamma}{\gamma}\tilde{C}$.

The representative agent consumes goods of both regions. Home consumption can be expressed by $C = \gamma k Y_A^{\nu} Y_B^{1-\nu}$, where $k \equiv (1-\nu)^{1-\nu}\nu^{\nu}$ and Y_i with $i = A, B$ denotes output of each region i.

The long-run steady state under flexible prices within the monetary union is given by

$$\bar{C} = \bar{Y}_A \tag{5.68}$$

where C is overall consumption of region A. This means that domestic output (which corresponds to the domestic real income) equals the demand for commodities of the representative agents living in region A in the long-run steady state. This assumption holds, because government spending is assumed to be zero, the government budget is balanced, and labor is completely immobile between the home and foreign region of the currency area. The assumption that labor is strongly immobile in Europe is stated in Proaño (2006) and by the speech of Gonzáles-Páramo (2005), to which we referred to in the introduction.

Applying the Taylor expansion of second order as explained at the beginning of this section, we can substitute for

$$\tilde{C} = \bar{Y}_A\left(\hat{Y}_A + \frac{1}{2}\hat{Y}_A^2 + \mathcal{O}(||\xi_A||^3)\right) \,, \tag{5.69}$$

where we make use of the definition $\hat{Y}_A \equiv \log(Y_A/\bar{Y}_A)$.

In the steady state, we have the following ralations: $\bar{m} = \gamma^{-1}(1-\gamma)\bar{Y}_A, \quad u_C = 1, \quad u_m =$

1, $\quad u_{CC} = -(1-\gamma)\bar{Y}_A^{-1}, \quad u_{mm} = -\gamma \bar{m}^{-1}, \quad u_{Cm} = \gamma \bar{Y}_A^{-1}$. If we insert these steady state expressions into (5.65), we obtain Equation (5.71) after some mathematical manipulations:

$$\tilde{u} = u_C \bar{Y}_A \left(\hat{Y}_A \left(1 + \gamma u_{C\xi} \tilde{\bar{\xi}}_A \right) + \frac{1}{2} \hat{Y}_A^2 \right) + \text{t.i.p.} + \mathcal{O}(||\xi_A||^3). \tag{5.70}$$

We define $q_1 \equiv -\frac{\gamma u_{C\xi} \tilde{\bar{\xi}}}{u_{CC} \bar{Y}_A}$ and obtain for the $u(.)$-part of the loss function the approximation

$$\tilde{u} = u_C \bar{Y} \left(\hat{Y}_A \left(1 + (1-\gamma) q_1 \right) + \frac{1}{2} \hat{Y}_A^2 \right) + \text{t.i.p.} + \mathcal{O}(||\xi_A||^3) \; . \tag{5.71}$$

Approximation of the Disutility of Labor Part

Second order Taylor expansion of the disutility of labor part for a representative agent j leads to

$$\tilde{v}_j = \tilde{Y}_A^j v_Y + \frac{1}{2} v_{YY} (\tilde{Y}_A^j)^2 + v_{Y\xi} \tilde{Y}_A^j \tilde{\xi}_A + \text{t.i.p.} + \mathcal{O}(||\xi_A||^3) \; . \tag{5.72}$$

Using the Taylor-approximation

$$\tilde{Y}_A^j = \bar{Y}_A^j \left(\hat{Y}_A^j + \frac{1}{2} (\hat{Y}_A^j)^2 + \mathcal{O}(||\xi_A||^3) \right) , \tag{5.73}$$

we can rewrite Equation (5.72) as

$$\tilde{v}_j = \bar{Y}_A^j v_Y \left[\hat{Y}_A^j \left(1 + \frac{v_{Y\xi} \tilde{\bar{\xi}}_A}{v_Y} \right) + \frac{(\hat{Y}_A^j)^2}{2} \left(1 + \frac{v_{YY} \bar{Y}_A^j}{v_Y} \right) \right] + \text{t.i.p.} + \mathcal{O}(||\xi_A||^3) \; . \tag{5.74}$$

Maximizing the utility function (5.64) and combining the first order conditions yields

$$v_Y = u_C (1-\kappa) \; , \tag{5.75}$$

where the marginal disutility of producing output v_Y is equal to

$$v_Y \equiv \frac{\partial v}{\partial Y_A^j} = d_A (Y_A^j)^{\beta - 1} \; . \tag{5.76}$$

Inserting the symmetric steady state output (under fully flexible prices) given by Equation (5.61) results

$$v_Y = \bar{d}_A \left(\frac{(\theta-1)(1-\bar{\tau}_A)}{\theta \bar{d}_A} \right) = \frac{(1-\bar{\tau}_A)(\theta-1)}{\theta} \ . \tag{5.77}$$

Analogously to the proceedings in Woodford (2003), we define a parameter κ as a size which "summarizes the overall distortions in the steady state output level as a result of both taxes and market power":

$$\kappa \equiv 1 - \frac{(1-\bar{\tau}_A)(\theta-1)}{\theta}. \tag{5.78}$$

Equation (5.75) states that the marginal utility from consumption equals the marginal disutility from labor. Using this condition with the disutility part (5.74) of an average representative agent, we obtain

$$\begin{aligned} \tilde{v} &= \frac{1}{n}\int_0^n v_j d_j \\ &= \bar{Y}_A u_C \left((1-\kappa)\left(1 + \frac{v_{\xi Y}\tilde{\xi}_A}{v_Y}\right) E(\hat{Y}_A^j) + \frac{1}{2}E((\hat{Y}_A^j)^2)(1-\kappa)\left(1+\frac{v_{YY}\bar{Y}_A}{v_Y}\right)\right) \\ &+ \text{t.i.p.} + \mathcal{O}(||\xi_A||^3) \ . \end{aligned} \tag{5.79}$$

Note that the expectations operator has to be used here, since agents are different from each other with respect to their ability to reset prices. The second derivative of v with respect to the steady state output can be expressed in terms of v_Y:

$$v_{YY} = (\beta-1)d_A(Y_A^j)^{\beta-2} = \frac{(\beta-1)\, v_Y}{Y_A^j} \ . \tag{5.80}$$

Solving for v_Y yields

$$v_Y = \frac{Y_A^j}{\beta-1} v_{YY} \ . \tag{5.81}$$

Replacing v_Y in Equation (5.79) yields

$$\tilde{v} = \bar{Y}_A u_C \left(\left[(1-\kappa) + (1-\kappa)(\beta-1)\frac{v_{Y\xi}\tilde{\xi}_A}{v_{YY}\bar{Y}_A} \right] E(\hat{Y}_j^A) \right.$$
$$\left. + E((\hat{Y}_A^j)^2)(1-\kappa)(1+\beta-1) \right) + \text{t.i.p.} + \mathcal{O}(||\xi_A||^3) \tag{5.82}$$
$$= \bar{Y}_A u_C \left(\left[1-\kappa + (\beta-1)\frac{v_{Y\xi}\tilde{\xi}_A}{v_{YY}\bar{Y}_A} \right] + \frac{\beta}{2}\left[(E\hat{Y}_A^j)^2 + \text{Var}\,\hat{Y}_A^j \right] \right)$$
$$+ \text{t.i.p.} + \mathcal{O}(||\xi_A||^3)\ . \tag{5.83}$$

To obtain Equation (5.83), we refer to Dixit and Lambertini (2003*a*), who assume that κ is small, meaning that it can be neglected when it enters multiplicatively. This is possible as $v_{YY}\tilde{\xi}_A$ is on average considerably smaller than $v_{YY}\bar{Y}_A$. Furthermore, we replace $E[(\hat{Y}_A^j)^2]$ by $(E[\hat{Y}_A^j])^2 + \text{Var}\,Y_A^j$.

A second-order Taylor approximation of the CES-aggregator Y_A of home goods leads to[15]

$$\hat{Y}_A = E\hat{Y}_A^j + \frac{1}{2}\left(1 - \frac{1}{\theta}\right)\text{Var}\,\hat{Y}_A^j + \mathcal{O}(||\xi_A||^3)\ . \tag{5.84}$$

Solving for $E\hat{Y}_A^j$, we have

$$E\hat{Y}_A^j = \hat{Y}_A - \frac{1}{2}\left(1 - \frac{1}{\theta}\right)\text{Var}\,\hat{Y}_A^j + \mathcal{O}(||\xi_A||^3), \tag{5.85}$$

and inserting into Equation (5.83) yields

$$\tilde{v} = \bar{Y}_A u_C \left(\left[1-\kappa + (\beta-1)q_2 \right] \hat{Y}_A + \frac{\beta}{2}\hat{Y}_A^2 + \frac{1}{2}\left[\beta - 1 + \frac{1}{\theta} \right] \text{Var}\,\hat{Y}_A^j \right)$$
$$+ \text{t.i.p.} + \mathcal{O}(||\xi_A||^3), \quad \text{where } q_2 \equiv -\frac{v_{Y\xi}\tilde{\xi}_A}{\bar{Y}_A v_{YY}}. \tag{5.86}$$

[15] See Woodford (2003, Chapter 6 and Appendix E), or Rotemberg and Woodford (1998).

Region-Specific Welfare Function

Subtracting (5.86) from (5.71) yields social welfare

$$U^A = -\frac{u_C \bar{Y}_A}{2}\left(\hat{Y}_A^2(\beta-1) - 2\hat{Y}_A\left[q_1(1-\gamma) + q_2(\beta-1) + \kappa\right]\right. \\ \left. + \frac{1+\theta(\beta-1)}{\theta}\mathrm{Var}\,\hat{Y}_A^j\right) + \text{t.i.p.} + \mathcal{O}(||\xi_A||^3)\ . \tag{5.87}$$

When log-linearizing the CES-aggregator over domestically produced differentiated goods, we obtain[16]

$$\hat{Y}_A^j = \log \bar{Y}_A - \theta(\log p^A(j) - \log P_A). \tag{5.88}$$

The variance of $\log \hat{Y}_A^j$ is given by

$$\mathrm{Var}(\hat{Y}_A^j) = \mathrm{Var}(\log \bar{Y}_A - \theta(\log p^A(j) - \log P_A)) = \theta^2 \mathrm{Var}(\log p^A(j)). \tag{5.89}$$

The last equation contains a very important finding of our model: As a domestic agent j only works in the home-region to produce the domestic good Y_A^j, and because the production of this good matters for his utility, he also cares only about the variability of this output. Hence, only the price level of domestically produced goods matters for the welfare of domestic agents. Of course, this finding is overturned, if high labor mobility is assumed, i.e. in our case, a representative agent j is free to produce the specific good in region B, region A, or in both regions. So far, for the EMU a high degree of labor mobility seems not to be backed by the data.

To apply the variance of domestic prices to the welfare equation for obtaining the inflation target, we need first to refer to the staggered price setting: We assume that a certain fraction Φ^A of firms (=producer-consumers) is not able to adjust the prices in response to a shock, while a fraction $(1-\Phi^A)$ can freely change their prices after a shock occurs. Then, inflation in region A is given by

$$\pi_A = \Phi^A \bar{\pi}_A + (1-\Phi^A)\tilde{\pi}^A \tag{5.90}$$

According to Dixit and Lambertini (2003*a*), we refer to the idea of Calvo-staggered pricing, which however reflects a dynamic setting (for details see Calvo 1983). Analogously to the procedure proposed by Dixit and Lambertini, we introduce a virtual discount factor η with $\eta < 1$ (which means that pseudo-future period utilities have a

[16] See also Woodford (2003, p.396).

lower weight than present utility). We, first, assume that η equals unity to explain the intuitional proceeding. In the case where prices are allowed to change, the optimal log price equals

$$\tilde{\pi}_A = (1-\Phi^A)\pi_A^j + \Phi^A \bar{\pi}_A$$

where π_A^j is the log steady-state deviation of the price that would be optimal if prices could be adjusted freely. The log price set by producer j is a sum of the weighted optimal price of producer j if prices were fully flexible, and the weighted price that maximizes the expected indirect utility if prices are to be fixed in future periods. The weights equal the probability of being able, $(1-\Phi^A)$, or not being able, Φ^A, to change the price in the following period(s).

Now we turn again to the discount factor $\eta < 1$: As already mentioned, the individuals place lower weight on future utilities. Therefore, the fact that the producer cannot change the price in future periods with a certain probability is expressed by a lower weight than the pure probability of future price setting (given by $\eta\Phi^i$) and a higher weight for the present period $(1-\eta\Phi^i)$. Hence, we obtain

$$\tilde{\pi}_A = (1-\Phi^A\eta)\pi_A^j + \Phi^A\eta\bar{\pi}_A. \tag{5.91}$$

Using the overall inflation rate in region A given by Equation (5.90) as well as the pseudo-intertemporal Equation (5.91) for the optimal inflation rate $\bar{\pi}_A$, we can express the inflation rate as a combination of variables that have a single period interpretation:

$$\pi_A = \rho^A\bar{\pi}_A + (1-\rho^A)\pi_A^j, \qquad \rho^A = \Phi^A[1+(1-\Phi^A)\eta]. \tag{5.92}$$

Again, $\bar{\pi}_A$ is the average inflation rate that arises when prices are set before the shocks occur, while π_A^j is the price that is optimal for the current period only, i. e. after uncertainty about the stochastic processes is resolved. With this single period representation we are able to apply the result of the first example in Woodford (2003, pp. 397f.) for our next steps. Firms that have to set prices before the shock materializes will set them identically according to the expected value of the optimal price for the period, i. e.

$$\log\bar{\pi}_A = E[\pi_A^j], \tag{5.93}$$

where E denotes the expectations operator. Subtracting the expectation of Equation

(5.92) from itself and noting that $E[\bar{\pi}_A] = \bar{\pi}_A$, we obtain

$$\pi_A - E[\pi_A] = \qquad (1-\rho^A)(\pi_A^j - E[\pi_A^j]) = (1-\rho^A)(\pi_A^j - \bar{\pi}_A). \tag{5.94}$$

We, now, combine Equations (5.89) and (5.94). The relation between the variance of $\log p^A(j)$ and the inflation goal in the representative agent's utility function is given by

$$\operatorname{Var}\log p^A(j) = \rho^A(1-\rho^A)(\pi_A^j - \bar{\pi}_A)^2 = \frac{\rho^A(1-\rho^A)}{(1-\rho^A)^2}(\pi_A - E[\pi_A])^2 = \frac{\rho^A(\pi_A - \bar{\pi}_A)^2}{1-\rho^A}. \tag{5.95}$$

Inserting relation (5.95) into (5.87) yields

$$\begin{aligned} U^A = -\frac{\bar{Y}u_C}{2}\Big(\hat{Y}_A^2(\beta-1) - 2\hat{Y}_A\left[q_1(1-\gamma) + q_2(\beta-1) + \kappa\right] \\ + \frac{\rho^A\theta(1+\theta(\beta-1))}{1-\rho^A}(\pi_A - \bar{\pi}_A)\Big) + \text{t.i.p.} + \mathcal{O}(||\xi_A||^3)\ . \end{aligned} \tag{5.96}$$

To obtain the output goal in the welfare function, we perform some mathematical manipulations:[17]

$$\begin{aligned} &\hat{Y}_A^2(\beta-1) - 2\hat{Y}_A\left[q_1(1-\gamma) + q_2(\beta-1)\right] - 2\kappa\hat{Y}_A \\ &= (\beta-1)\left(\hat{Y}_A^2 - 2\hat{Y}_A\frac{q_1(1-\gamma)+q_2(\beta-1)}{\beta-1} - \frac{2\hat{Y}_A}{\beta-1}\kappa\right)\ . \end{aligned} \tag{5.97}$$

The *natural* rate of output in region A, $\hat{Y}_A^n$, which materializes under flexible prices in the setting of monopolistic competition, is given by a log-linearization of Equation (5.61). It can be expressed in terms of the region-specific variables q_1 and q_2:[18]

$$\hat{Y}_A^n = \frac{(1-\gamma)q_1 + (\beta-1)q_2}{\beta-1}\ . \tag{5.98}$$

[17] Note that q_1 and q_2 are region specific notations, as both contain first and second derivatives of a representative household's utility, who lives in region A.

[18] To see this, note that the log-linearized version of Equation (5.61) reads $\hat{Y}_A^n = \frac{-1}{\beta-1}(\hat{\xi}_i + \frac{\bar{\tau}_i}{1-\bar{\tau}_i}\hat{\tau}_i)$. Rewriting the terms in the last parentheses in absolute deviations from steady state, we obtain $\hat{Y}_A^n = \frac{-1}{\beta-1}(\frac{\xi_i}{\bar{\xi}_i} - \frac{1-\tau_i}{1-\bar{\tau}_i}) = \frac{-1}{\beta-1}(Y_i/\bar{Y}_i)$. Given our assumption $u_{C,\xi} = 0$, Equation (5.98) is merely a more general notation for the equation given in this footnote.

Using this result and adding terms from the t.i.p.-part, we can rewrite (5.97) as

$$
\begin{aligned}
&(\beta-1)\left(\hat{Y}_A^2 - 2\hat{Y}_A\hat{Y}_A^n + (\hat{Y}_A^n)^2 - 2\frac{\hat{Y}_A\kappa}{\beta-1} - 2\frac{\hat{Y}_A^n\kappa}{\beta-1} + \frac{\kappa^2}{(\beta-1)^2}\right) + 2\frac{\hat{Y}_A^n\kappa}{\beta-1} - \frac{\kappa^2}{(\beta-1)^2} \\
&= (\beta-1)\left((\hat{Y}_A - \hat{Y}_A^n)^2 - 2(\hat{Y}_A - \hat{Y}_A^n)\hat{y}_A + (\hat{y}_A)^2\right) + \text{t.i.p.}\,, \qquad (5.99)
\end{aligned}
$$

where we define $\hat{y}_A$ as the log deviation of steady state level given by Equation (5.61), from the steady state of efficient output given by Equation (5.63), evaluated at $\xi_i = \bar{\xi}_i$. Formally,

$$
\begin{aligned}
\hat{y}_A &= \log\left(\frac{\bar{Y}_A}{\bar{Y}_A^{\text{eff}}}\right) = \log\left[\frac{\left(\frac{(\theta-1)(1-\bar{\tau}_A)}{\theta\bar{\xi}_A}\right)^{\frac{1}{\beta-1}}}{\left(\frac{\theta-1}{\theta\bar{\xi}_A-1}\right)^{\frac{1}{\beta-1}}}\right] = \frac{1}{\beta-1}\log\left(\frac{(\theta-1)(1-\bar{\tau}_A)}{\theta}\right) \\
&\approx \frac{1}{\beta-1}\left(\frac{(\theta-1)(1-\bar{\tau}_A)}{\theta} - 1\right) = -\frac{\kappa}{\beta-1}, \qquad (5.100)
\end{aligned}
$$

where $\bar{\xi}_A = 1$, the approximation holds only for values of $(\theta-1)(1-\bar{\tau}_A)\theta^{-1}$ close to one, and the last equation uses the notation of Equation (5.78). The variable $\hat{y}_A$ summarizes the overall distortions in steady state output. Inserting (5.99) into (5.96) yields

$$
\begin{aligned}
U^A = &-\frac{\bar{Y}u_C}{2}\left(\frac{\rho^A}{1-\rho^A}\theta\left[1+\theta(\beta-1)\right](\pi_A - \bar{\pi}_A)^2 + (\beta-1)(\hat{Y}_A - \hat{Y}_A^n - \hat{y}_A)^2\right) \\
&+ \text{t.i.p.} + \mathcal{O}(||\xi_A||^3)\,. \qquad (5.101)
\end{aligned}
$$

Rearranging terms, and replacing $\hat{Y}_i$ by y_i and $\hat{Y}_A^n + \hat{y}_A$ by $\bar{\bar{y}}_A$ yields the welfare function for home region A:

$$
L_A = \frac{1}{2}\left((\pi_A - \bar{\pi}_A)^2 + \theta_A(y_A - \bar{\bar{y}}_A)^2\right) + \text{t.i.p.} + \mathcal{O}(||\xi_A||^3)\,, \qquad (5.102)
$$

where $\theta_A \equiv (1-\rho^A)(\beta-1)[\rho^A\theta(1+\theta(\beta-1))]^{-1}$ is the relative weight of the output goal. Analogously, the welfare function for region B equals

$$
L_B = \frac{1}{2}\left((\pi_B - \bar{\pi}_B)^2 + \theta_B(y_B - \bar{\bar{y}}_B)^2\right) + \text{t.i.p.} + \mathcal{O}(||\xi_A||^3)\,. \qquad (5.103)
$$

Remark that our notation in terms of percentage changes in output differs slightly from the one with an “output gap" typically used in the literature. The reason for this is twofold. First, we want to express the variables in our policy analysis throughout in terms of inflation and output. Second, we prefer the explicit notation of observables rather than using the empirically less precise concept of an output gap.

5.1.8 Welfare Criterion for Monetary Policy

In this subsection, we pose the target function for a welfare-optimizing monetary authority, by taking up the solution of the previous subsection. The common central bank maximizes the utility for all agents living in the monetary union, i.e.

$$U(C, M/P, Y) = \int_0^n U^j(C_j^A, M_{j,A}/P_A, Y_{j,A})\,dj + \int_n^1 U^j(C_j^B, M_{j,B}/P_B, Y_{j,B})\,dj\ . \quad (5.104)$$

Due to the assumption of the immobility of labor between the two regions, the welfare maximization follows the same principle as for the single fiscal policies. Hence, we can write the monetary welfare function as

$$\begin{aligned} L_M = \frac{1}{2} \big[& n\big((\pi_A - \bar{\pi}_A)^2 + \theta_A^M (y_A - \bar{\bar{y}}_A)^2\big) \\ & + (1-n)\big((\pi_B - \bar{\pi}_B)^2 + \theta_B^M (y_B - \bar{\bar{y}}_B)^2\big)\big] + \text{t.i.p.} + \mathcal{O}(||\xi_A||^3)\ . \end{aligned} \quad (5.105)$$

A monetary welfare-function of this type is used in e.g. Pappa (2004).

5.2 Technical Appendix to Chapter 3

5.2.1 Data Appendix

In our empirical analysis, we use quarterly data from 1980:1 to 2004:4 for the U.S., U.K., Japan and Germany. The series are, with name, source and mnemonic in EcoWin:

- For the U.S.:
  - United States, House Prices, NATIONAL, Index; The Office for Federal Housing Enterprise Oversight; ew:usa11750
  - United States, National Income Account, Gross Domestic Product, Overall, Total, Current Prices, USD; U.S. Department of Commerce; ew:usa01151
  - United States, NYSE, Composite Index, End of Period, USD; EcoWin AB; ew:usa15755
  - United States, Consumer Price Index, All items, Total; OECD; oecd:usa_cpaltt01_ixobq
  - United States, Working-age population; OECD; oe:usa_poptq
- For the U.K.:
  - United Kingdom, House Prices, Nationwide, United Kingdom, all properties, Index; Nationwide(?); ew:gbr03621
  - United Kingdom, Expenditure Approach, Gross Domestic Product, Total, Current Prices, GBP; Office for National Statistics (ONS); ew:gbr01022
  - United Kingdom, FTSE, 30, Ordinary Share Index, Average, GBP; FTID; ew:gbr15105
  - United Kingdom, Consumer Price Index, All items, Total; OECD; oecd:gbr_cpaltt01_ixobq
  - United Kingdom, Working-age population; OECD; oe:gbr_poptq
- For Japan:
  - Japan, Consumer Prices, Nationwide, subgroup, housing excluding imputed rent, Index; Ministry of Internal Affairs and Communications; ew:jpn11802

  - Japan, Expenditure Approach, Gross Domestic Product, Total, Current Prices, JPY; Economic and Social Research Institute (ESRI); ew:jpn01320
  - Japan, Nikkei, 225, Index, End of Period, JPY; EcoWin AB; ew:jpn15005
  - Japan, Consumer Price Index, All items, Total; OECD; oecd:jpn_cpaltt01_ixobq
  - Japan, Working-age population; OECD; oe:jpn_poptq

- For Germany:
  - Germany, House Prices, Total, excl. cellar; Federal Statistical Office Germany; ew:deu11500
  - Germany, Expenditure Approach, Gross Domestic Product, Total (linked), Current Prices, EUR; EcoWin AB; ew:deu01994
  - Germany, Deutsche Boerse, DAX 30, Index, Total Return, End of Period, EUR; EcoWin AB; ew:deu15005
  - Germany, Consumer Price Index, All items, Total; OECD; oecd:deu_cpaltt01_ixobq
  - Germany, Working-age population; OECD; oe:deu_poptq, together with the semiannual series oe:deu_popts for pre-1991 data, averaged to quarterly frequency

The series are - if necessary - transformed to quarterly frequency, deseasonalized and deflated using the CPI. The real GDP is divided by the working age population to obtain the appropriate measure for real GDP per capita. We then index the series so that the average of 1980-1995 equals zero, and take (natural) logarithms. Because of the observed long cycles in house prices, other detrending methods like the Hodrick-Prescott filter lead to different results, especially for the correlations between house prices and stock prices.[19]

[19]See European Central Bank (2003) and International Monetary Fund (2004) for discussions on possible detrending procedures for house prices, and Canova (1998) for a detailed analysis of the effects of different detrending methods.

5.3 Technical Appendix to Chapter 4

5.3.1 Steady State

For the derivation of the nonstochastic perfect foresight steady state, I assume without loss of generality that steady state domestic technology $A = 1$. For notational simplicity, I omit a variable's time subscript to denote its steady state. In the steady state, prices are flexible and markups are constant. In connection with firms' pricing derived in Section 5.3.5, this implies

$$MC = \frac{MC^n}{P_H} = \frac{(1-\tau)W}{P_H} = \frac{\varepsilon - 1}{\varepsilon}. \tag{5.106}$$

Plugging this result in the household's intratemporal first-order condition gives

$$C^\sigma N^\varphi = \frac{W}{P} \tag{5.107}$$

$$\Leftrightarrow \quad C^\sigma Y^\varphi = \frac{\varepsilon - 1}{\varepsilon} \frac{1}{1-\tau} \frac{P_H}{P}, \tag{5.108}$$

where the latter equation used the steady state relationship $Y = AN = N$. From the risk sharing condition (4.31) we obtain

$$C = \vartheta Y^* \mathcal{Q}^{\frac{1}{\sigma}}, \tag{5.109}$$

using $C^* = Y^*$. Replacing C in (5.107) by equation (5.109) leads to

$$Y = \left(\frac{\frac{\varepsilon-1}{\varepsilon} \frac{P_H}{P}}{(1-\tau)(\vartheta Y^*)^\sigma \mathcal{Q}} \right)^{\frac{1}{\varphi}} \tag{5.110}$$

$$= \left(\frac{1 - \frac{1}{\varepsilon}}{1-\tau} \right)^{\frac{1}{\varphi}} \mathcal{S}^{-\frac{1}{\varphi}} (\vartheta Y^*)^{-\frac{\sigma}{\varphi}}, \tag{5.111}$$

where the second line replaced the price ratio and the real exchange rate by the terms of trade, along

$$\mathcal{Q} = \frac{\mathcal{E} P^*}{P} = \mathcal{S} \frac{P_H}{P}, \qquad \text{as} \qquad \mathcal{S} = \frac{(1-\kappa) P_F}{P_H} = \frac{\mathcal{E} P_F^*}{P_H} = \frac{\mathcal{E} P^*}{P_H}, \tag{5.112}$$

see Equations (4.11), (4.12), (4.15) and (4.17).

Furthermore, transforming the market clearing condition (4.25) gives rise to a second equation linking domestic output to foreign output and the terms of trade:

$$
\begin{aligned}
Y &= C_H + \frac{1}{1-\kappa} C_H^* && (5.113)\\
&= (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C + \frac{\alpha^*}{1-\kappa}\left(\frac{P_H}{\mathscr{E}P^*}\right)^{-\eta} C^* && (5.114)\\
&= (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C + \frac{\alpha^*}{1-\kappa}\mathscr{S}^{\eta} Y^* && (5.115)\\
&= \left[(1-\alpha)\left(\frac{P_H}{P}\right)^{\frac{1}{\sigma}-\eta}\mathscr{S}^{\frac{1}{\sigma}} + \frac{\alpha}{1-\kappa}\mathscr{S}^{\eta}\right]\vartheta Y^*, && (5.116)
\end{aligned}
$$

where α^* is replaced by $\alpha\vartheta$, where $\vartheta = \frac{C_0}{C_0^*}$ denotes initial conditions of the model. Equations (5.111) and (5.116) together determine the terms of trade and domestic output as functions of world output. The unique solution for the terms of trade is given by $\mathscr{S} = \frac{(1-\kappa)P_F}{P_H} = 1$. This result can be used to simplify the CPI Equation (4.6)

$$
\begin{aligned}
\bar{P}^{1-\eta} &= (1-\alpha)\bar{P}_H^{1-\eta} + \alpha\bar{P}_F^{1-\eta} && (5.117)\\
&= [1-\alpha+\alpha(1-\kappa)^{\eta-1}]\bar{P}_H^{1-\eta} && (5.118)\\
&= [\alpha+(1-\alpha)(1-\kappa)^{1-\eta}]\bar{P}_F^{1-\eta} && (5.119)
\end{aligned}
$$

and to solve it for the steady state ratios:

$$
\begin{aligned}
\frac{P_H}{P} &= \left[1-\alpha+\alpha(1-\kappa)^{\eta-1}\right]^{\frac{1}{\eta-1}} \equiv \Phi_{PHP} && (5.120)\\
\frac{P_F}{P} &= \left[\alpha+(1-\alpha)(1-\kappa)^{1-\eta}\right]^{\frac{1}{\eta-1}} \equiv \Phi_{PFP}. && (5.121)
\end{aligned}
$$

Notice that these ratios are equal to unity if trade costs are zero, $\kappa = 0$, or if the substitution elasticity is $\eta = 1$. With this in mind, Equations (5.111) and (5.116) simplify to

$$
Y = \left(\frac{1-\frac{1}{\varepsilon}}{1-\tau}\right)^{\frac{1}{\varphi}} (\vartheta Y^*)^{-\frac{\sigma}{\varphi}} = \Phi_{SS1}(\vartheta Y^*)^{-\frac{\sigma}{\varphi}} \tag{5.122}
$$

and

$$
Y = \left[(1-\alpha)\Phi_{PHP}^{\frac{1}{\sigma}-\eta} + \frac{\alpha}{1-\kappa}\right]\vartheta Y^* = \Phi_{SS2}\vartheta Y^*. \tag{5.123}
$$

The solution to this system is given by

$$Y^* = \frac{1}{\vartheta}\left(\frac{\Phi_{SS1}}{\Phi_{SS2}^{\varphi}}\right)^{\frac{1}{1+\sigma}} \tag{5.124}$$

and

$$Y = \Phi_{SS2}\vartheta Y^*. \tag{5.125}$$

Some remarks are in order. First, in the case of zero trade costs, $\Phi_{SS2} = 1$ and $Y = \vartheta Y^*$, as in GM. For positive trade costs (and $\eta > 1$), the relative size of domestic output increases, as $\Phi_{SS2} > 1$. Trade costs decrease the demand for imports and increase domestic production. As the small open economy is by definition more open, this effect is more pronounced for the small open economy. Hence, the size effect on the output ratio Y/Y^*.

Second, for positive trade costs, $\Phi_{PFP} > 1 > \Phi_{PHP}$, i. e., the price index of imports is higher than the average price index, reflecting transport costs.

Third, the steady state real exchange rate $\mathcal{Q} = \Phi_{PHP}\mathcal{S}$ is unity under zero trade costs, but smaller than unity for $\kappa > 0$.

Fourth, inspecting Equation (5.109), steady state consumption in the small open economy equals domestic output for zero trade costs. For positive trade costs, steady state consumption becomes smaller than steady state output. At first glance, this might seem unreasonable, as it suggests that the small open economy does not spend all its income. However, this is not the case, as “some portion of the traded good dissipates in transit".[20]

Fifth, trade costs also influence steady state net exports. Nominal net exports are given by Equation (4.22). In steady state, this reads

$$NX = Y - PC/P_H = Y - C/\Phi_{PHP}. \tag{5.126}$$

As in GM steady state net exports are zero for $\kappa = 0$, but they are negative for positive trade costs, where $\Phi_{PHP} < 1$.

[20]See Obstfeld and Rogoff (1996, p. 251).

5.3.2 Log-Linearization of the CPI Equation

I linearly approximate the domestic CPI, as given by Equation (4.6) around the steady state, where $\bar{P}_H = (1-\kappa)\bar{P}_F$. Rewriting the CPI equation as

$$P_t^{1-\eta} = (1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}, \tag{5.127}$$

it is straightforward to log-linearize this equation to get

$$p_t = (1-\alpha)\left(\frac{\bar{P}_H}{\bar{P}}\right)^{1-\eta} p_{H,t} + \alpha\left(\frac{\bar{P}_F}{\bar{P}}\right)^{1-\eta} p_{F,t}, \tag{5.128}$$

where small letters denote log deviations from the steady state. The constant steady state ratios P_H/P and P_F/P are derived in Section 5.3.1 of the appendix, they are given in Equations (5.120) and (5.121). Plugging them in Equation (5.128) yields

$$\begin{aligned} p_t &= \frac{1-\alpha}{1-\alpha+\alpha(1-\kappa)^{\eta-1}} p_{H,t} && + \frac{\alpha}{\alpha+(1-\alpha)(1-\kappa)^{1-\eta}} p_{F,t} \\ &= \left(1 - \frac{\alpha}{\alpha+(1-\alpha)(1-\kappa)^{1-\eta}}\right) p_{H,t} && + \frac{\alpha}{\alpha+(1-\alpha)(1-\kappa)^{1-\eta}} p_{F,t} \\ &= (1-\alpha')p_{H,t} + \alpha' p_{F,t}. \end{aligned} \tag{5.129}$$

The last equation is Equation (4.19) in the text. Notice that the coefficients $1-\alpha'$ and α' sum up to one like $1-\alpha$ and α in GM, they actually coincide with them in the case of zero trade costs $\kappa = 0$. These coefficients show the relative importance of changes in domestic producer prices and import prices for changes in the CPI. In GM, the baseline value $\alpha = 0.4$ implies that import prices affect the CPI by 40 percent. In my baseline calibration with substitution elasticity $\eta = 1.5$ and trade costs $\kappa = 0.25$, this effect is reduced to 36.6 percent as a result of the trade reducing costs. Notice, however, that trade costs only influence the CPI if the international substitution elasticity is non-unitary. Using the same value for this elasticity as for the intranational substitution elasticity, i.e., setting $\eta = \varepsilon = 6$, the effect of imports on the CPI is reduced by more than one half, to 13.7 percent. The higher the substitutability between domestic and foreign goods, the easier it is to replace trade cost affected imports by domestically produced goods. Finally, in the OR baseline of $\eta = 6$, $\kappa = 0.25$ and $\alpha = 0.5$ (no home bias), the effect of imports is again strongly reduced to 19.2 percent.

5.3.3 Log-Linearization of Net Exports Equation

Nominal net exports are given by

$$P_{H,t}NX_t = P_{H,t}Y_t - P_tC_t. \tag{5.130}$$

As Section 5.3.1 shows, the steady state implies $NX = Y - PC/P_H = Y - C/\Phi_{PHP}$, which could be zero. Hence, log deviations of net exports around steady state cannot be defined in the usual way. Instead, define

$$nx_t \equiv \frac{NX_t - NX}{Y} \tag{5.131}$$

to be the percentage deviation of net exports from steady state in terms of domestic steady state GDP. Rewriting Equation (5.130), we have

$$NX_t = Y_t - \frac{P_t}{P_{H,t}}C_t \tag{5.132}$$

$$\Leftrightarrow \quad Y nx_t + NX = Y(1+y_t) - \frac{PC}{P_H}(1 + p_t - p_{H,t} + c_t) \tag{5.133}$$

$$\Leftrightarrow \quad nx_t = y_t - \frac{PC}{P_HY}(p_t - p_{H,t} + c_t) \tag{5.134}$$

$$= y_t - \frac{PC}{P_HY}(c_t + \alpha' s_t), \tag{5.135}$$

where the last equation, obtained using Equation (4.20), is Equation (4.23) in the main text. The steady state ratio $\frac{PC}{P_HY}$ can be solved for parameters using equations (5.109), (5.125) and $\mathcal{Q} = \Phi_{PHP}\mathcal{S}$. One then gets $\frac{PC}{P_HY} = \Phi_{PHP}^{\frac{1}{\sigma}-1}\Phi_{SS2}^{-1}$. Notice that in the case of zero trade costs, steady state nominal net exports are zero, and hence $P_HY = PC$, or $\Phi_{PHP} = \Phi_{SS2} = 1$, so that one obtains the GM result

$$nx_t = y_t - c_t - \alpha' s_t.$$

5.3.4 Derivation of the Risk Sharing Condition Equation

Equating the domestic Euler Equation (4.29) and its foreign analog given in footnote 11, we have

$$C_t = C_t^* \left(\frac{\mathcal{E}_t P_t^*}{P_t}\right)^{\frac{1}{\sigma}} E_t\left[\frac{C_{t+1}}{C_{t+1}^*}\left(\frac{P_{t+1}}{\mathcal{E}_{t+1}P_{t+1}^*}\right)^{\frac{1}{\sigma}}\right]. \tag{5.136}$$

Using the definition of the real exchange rate, $\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$, this may be rewritten as

$$\frac{C_t}{C_t^* \mathcal{Q}_t^{\frac{1}{\sigma}}} = E_t \left[\frac{C_{t+1}}{C_{t+1}^* \mathcal{Q}_{t+1}^{\frac{1}{\sigma}}} \right]. \tag{5.137}$$

Iterating this equation backwards and assuming that the period zero real exchange rate is at its steady state, $\mathcal{Q}_t = 1$, and denoting initial conditions $\frac{C_0}{C_0^*} = \vartheta$, we get

$$\frac{C_t}{C_t^* \mathcal{Q}_t^{\frac{1}{\sigma}}} = \vartheta, \tag{5.138}$$

which, multiplied by the denominator, is Equation (4.31) in the text.

5.3.5 Derivation of the Price Setting Rule Equation

A representative firm i faces the following maximization problem:

$$\max_{\overline{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} [Y_{t+k} (\overline{P}_{H,t} - MC_{t+k}^n)] \} , \tag{5.139}$$

subject to the demand function. Demand for domestic good i is the sum of demand from the small open economy and the world economy. But as a fraction κ of the good melts away in the trade process, consumption abroad is only $1-\kappa$ of what was meant for export of good i. From the market clearing Equation (4.25), we obtain for good i

$$C_{H,t}^*(i) = (1-\kappa)[Y_t(i) - C_{H,t}(i)]. \tag{5.140}$$

Hence, demand can be written as

$$Y_t^d(i) = C_{H,t}(i) + \frac{1}{1-\kappa} C_{H,t}^*(i) \tag{5.141}$$

$$= \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} + \left(\frac{P_{H,t}^*(i)}{P_{H,t}^*} \right)^{-\varepsilon} \frac{1}{1-\kappa} C_{H,t}^* \tag{5.142}$$

$$= \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left(C_{H,t} + \frac{1}{1-\kappa} C_{H,t}^* \right), \tag{5.143}$$

where I have made use of Equation (4.26) in the second line and of the nominal exchange rate definition in the third line, where trade costs cancel each other out in the numerator and in the denominator. At date $t+k$, good i production is not bigger than its demand. Replacing the individual price $P_{H,t}(i)$ by the newly set price $\overline{P}_{H,t}$, the constraint to the maximization problem reads

$$Y_{t+k}(i) \leq \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} (C_{H,t+k} + \frac{1}{1-\kappa} C^*_{H,t+k}) \equiv Y^d_{t+k}(\overline{P}_{H,t}). \tag{5.144}$$

Each firm sets the same price in equilibrium, so the index i can be dropped. As equality holds in the optimum, one can replace Y_{t+k} in the maximization problem by the constraint given in Equation (5.144). Multiplying by $\bar{P}_{H,t}$, dividing by $1-\varepsilon$ and reinserting Y_{t+k}, the according first order condition looks as follows:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k} (\overline{P}_{H,t} - \frac{\varepsilon}{\varepsilon-1} MC^n_{t+k}) \right\} = 0 \,. \tag{5.145}$$

Using the household's Euler Equation (4.29) and the fact that $E_t(Q_{t,t+1}) = \frac{1}{R_t}$, one can replace $E_t(Q_{t,t++k})$ by $\beta^k \left(\frac{C_t}{C_{t+k}}\right)^{\sigma} \frac{P_t}{P_{t+k}}$. Dividing by the period t terms results in

$$\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{1}{P_{t+k} C^{\sigma}_{t+k}} Y_{t+k} (\overline{P}_{H,t} - \frac{\varepsilon}{\varepsilon-1} MC^n_{t+k}) \right\} = 0 \,. \tag{5.146}$$

In preparation for log-linearization, split up the difference and notice that $MC_{t+k} \equiv \frac{MC^n_{t+k}}{P_{H,t+k}}$:

$$\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\bar{P}_{H,t} Y_{t+k}}{P_{t+k} C^{\sigma}_{t+k}} \right\} = \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\varepsilon Y_{t+k} MC_{t+k} P_{H,t+k}}{(\varepsilon-1) P_{t+k} C^{\sigma}_{t+k}} \right\}. \tag{5.147}$$

Next I log-linearize around the zero inflation, perfect foresight, balanced trade steady state. For this, notice that at the steady state, $\bar{P}_{H,t} = P_{H,t+k}$,and $MC_{t+k} = \frac{\varepsilon-1}{\varepsilon}$. Using small letters to denote percentage deviations around steady state, we get

$$\begin{aligned} &\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \bar{p}_{H,t} + y_{t+k} - p_{t+k} - \sigma c_{t+k} \right\} \\ &= \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ y_{t+k} + \widehat{mc}_{t+k} + p_{H,t+k} - p_{t+k} - \sigma c_{t+k} \right\}, \end{aligned} \tag{5.148}$$

where I have already factored out and divided by the steady state values. Notice that I have written $\widehat{mc}_t$ instead of mc_t, to keep notation consistent with GM and Galí and Monacelli (2005b), who use $mc_t \equiv \log MC_t$, $mc^n_t \equiv \log MC^n_t$ and $\widehat{mc}_t \equiv mc_t - \overline{mc}$, where $\overline{mc} = \log \overline{MC} = \log \frac{\varepsilon-1}{\varepsilon} \equiv -\mu$ is the steady state real marginal cost. Simplifying the last equation using $\sum_{k=0}^{\infty} (\beta\theta)^k = 1/(1-\beta\theta)$ results in

$$\bar{p}_{H,t} = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \widehat{mc}_{t+k} + p_{H,t+k} \right\}. \tag{5.149}$$

Rewriting $\widehat{mc}_t^n = \widehat{mc}_t^n + p_{H,t}$, this can be transformed to

$$\overline{p}_{H,t} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\widehat{mc}_{t+k}^n\}\ , \tag{5.150}$$

which is Equation (4.34) in Section 4.2.3.

5.3.6 Derivation of the Inflation Dynamics Equation

In the Calvo pricing scheme, the domestic price level given in equation (4.7) can be rewritten as the combination of previous period's price and the newly set price:

$$P_{H,t} = [\theta P_{H,t-1}^{1-\varepsilon} + (1-\theta)\overline{P}_{H,t}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}\ . \tag{5.151}$$

Log-linearizing this equation around a zero inflation steady state results in

$$p_{H,t} = \theta p_{H,t-1} + (1-\theta)\bar{p}_{H,t}. \tag{5.152}$$

From the previous paragraph, notice that Equation (5.150) can be rewritten as a first-order difference equation in $p_{H,t}$. Leading the equation by one, taking conditional expectations and multiplying by $\beta\theta$ and subtracting this from the original equation gives

$$\bar{p}_{H,t} = (1-\beta\theta)(\widehat{mc}_t^n) + \beta\theta E_t\{\bar{p}_{H,t+1}\}. \tag{5.153}$$

Now, multiply this equation by $(1-\theta)$. Then, replace $(1-\theta)\bar{p}_{H,t}$ by making use of Equation (5.152), both at date t and date $t+1$. This results in

$$p_{H,t} - \theta p_{H,t-1} = (1-\theta)(1-\beta\theta)(\widehat{mc}_t^n) + \beta\theta E_t\{p_{H,t+1} - \theta p_{H,t}\}. \tag{5.154}$$

Using $\widehat{mc}_t^n = \widehat{mc}_t + p_{H,t}$ and simplifying, we obtain

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda(\widehat{mc}_t), \quad \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}, \tag{5.155}$$

which is the small open economy part of Equation (4.35) in Section 4.2.3. The world inflation is determined analogously.

5.3.7 Derivation of the Canonical Representation

In this section, I derive the dynamic IS equation and the New Keynesian Phillips Curve (NKPC) for the world economy and the small open economy.

Writing the foreign analog of the household's log-linear Euler Equation (4.30) in terms of foreign currency, using the market clearing condition (4.24), one obtains a difference equation for world output:

$$y_t^* = E_t\{y_{t+1}^*\} - \frac{1}{\sigma}(r_t^* - E_t\{\pi_{t+1}^*\}) \, . \tag{5.156}$$

For the small open economy, an analog can be achieved in eight steps: First, I write down the market clearing condition (4.25) for a domestically produced good i. Then, I use the demand functions (4.26) and (4.27) as well as its world analogs. Here, notice that under producer currency pricing the substitution elasticity for domestically produced goods has to be considered. Third, I replace total consumption in the small open economy by world output, following Equation (4.31):

$$\begin{aligned}
Y_t(i) &= C_{H,t}(i) + \frac{1}{1-\kappa} C_{H,t}^*(i) && (5.157)\\
&= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} (1-\alpha) C_t + \left(\frac{P_{H,t}}{\mathcal{E}_t P_t^*}\right)^{-\eta} \frac{\alpha^*}{1-\kappa} Y_t^* \right] && (5.158)\\
&= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \vartheta Y_t^* \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} (1-\alpha) \mathcal{Q}_t^{\frac{1}{\sigma}} + \left(\frac{P_{H,t}}{\mathcal{E}_t P_t^*}\right)^{-\eta} \frac{\alpha}{1-\kappa} \right] . && (5.159)
\end{aligned}$$

In the fourth step, define domestic output like consumption as in Equation (4.3) to be

$$Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{5.160}$$

and plug Equation (5.159) into this definition:

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} d_i\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \left[\int_0^1 \left\{\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} \vartheta Y_t^* \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}(1-\alpha)\mathcal{Q}_t^{\frac{1}{\sigma}} + \left(\frac{P_{H,t}}{\mathcal{E}_t P_t^*}\right)^{-\eta} \frac{\alpha}{1-\kappa}\right]^{\frac{\epsilon-1}{\epsilon}} \right\} di\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \left[\left[\vartheta Y_t^* \left\{\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}(1-\alpha)\mathcal{Q}_t^{\frac{1}{\sigma}} + \mathcal{S}^{\eta}\frac{\alpha}{1-\kappa}\right\}\right]^{\frac{\epsilon-1}{\epsilon}} P_{H,t}^{\epsilon-1}\int_0^1 P_{H,t}(i)^{1-\epsilon} di\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \left[\left[\vartheta Y_t^* \left\{\left(\frac{P_{H,t}\mathcal{Q}_t}{\mathcal{E}_t P_t^*}\right)^{-\eta}(1-\alpha)\mathcal{Q}_t^{\frac{1}{\sigma}} + \mathcal{S}^{\eta}\frac{\alpha}{1-\kappa}\right\}\right]^{\frac{\epsilon-1}{\epsilon}} P_{H,t}^{\epsilon-1}\int_0^1 P_{H,t}(i)^{1-\epsilon} di\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \left[\left[\vartheta Y_t^* \left\{\mathcal{S}^{\eta}\mathcal{Q}_t^{-\eta}(1-\alpha)\mathcal{Q}_t^{\frac{1}{\sigma}} + \mathcal{S}^{\eta}\frac{\alpha}{1-\kappa}\right\}\right]^{\frac{\epsilon-1}{\epsilon}} P_{H,t}^{\epsilon-1}\int_0^1 P_{H,t}(i)^{1-\epsilon} di\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \vartheta Y_t^* \mathcal{S}^{\eta}\left[(1-\alpha)\mathcal{Q}_t^{\frac{1}{\sigma}-\eta} + \frac{\alpha}{1-\kappa}\right] P_{H,t}^{\epsilon}\left[\int_0^1 P_{H,t}(i)^{1-\epsilon} di\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \vartheta Y_t^* \mathcal{S}^{\eta}\left[(1-\alpha)\mathcal{Q}_t^{\frac{1}{\sigma}-\eta} + \frac{\alpha}{1-\kappa}\right] P_{H,t}^{\epsilon}(P_{H,t}^{1-\epsilon})^{\frac{\epsilon}{\epsilon-1}} \\
&= \vartheta Y_t^* \mathcal{S}^{\eta}\left[(1-\alpha)\mathcal{Q}_t^{\frac{1}{\sigma}-\eta} + \frac{\alpha}{1-\kappa}\right]. \qquad (5.161)
\end{aligned}
$$

Notice that I have made use of $\frac{P_{H,t}}{P_t} = \frac{P_{H,t}\mathcal{Q}_t}{\mathcal{E}_t P_t^*}$ and $\frac{P_{H,t}}{\mathcal{E}_t P_t^*} = \mathcal{S}_t^{-1}$ during the calculations.

Step five is log-linearization around the steady state, following the principle $Y_t = Ye^{y_t} \approx Y(1+y_t)$. Simplifying,

$$Y_t = \vartheta Y_t^* \mathcal{S}_t^{\eta}(1-\alpha)\mathcal{Q}_t^{\frac{1}{\sigma}-\eta} + \frac{\alpha\vartheta}{1-\kappa} Y_t^* \mathcal{S}_t^{\eta}\,, \qquad (5.162)$$

this is well approximated by

$$
\begin{aligned}
Y(1+y_t) = {} & \vartheta Y^* \mathcal{S}^{\eta}(1-\alpha)\mathcal{Q}^{\frac{1}{\sigma}-\eta}[1 + y_t^* + \eta s_t + (\frac{1}{\sigma}-\eta)q_t] \\
& + \frac{\alpha\vartheta}{1-\kappa} Y^* \mathcal{S}^{\eta}(1 + y_t^* + \eta s_t). \qquad (5.163)
\end{aligned}
$$

After subtracting the steady state $Y = \Phi_{SS2}\vartheta Y^*$ given in Equation (5.123), this becomes

$$\begin{aligned} y_t &= \Phi_{SS2}^{-1}\Phi_{PHP}^{\frac{1}{\sigma}-\eta}(1-\alpha)[y_t^* + \eta s_t + (\frac{1}{\sigma}-\eta)q_t] + \Phi_{SS2}^{-1}\frac{\alpha}{1-\kappa}(y_t^* + \eta s_t) \\ &= y_t^* + \eta s_t + \left(1 - \frac{\alpha}{(1-\kappa)\Phi_{SS2}}\right)(\frac{1}{\sigma}-\eta)q_t \\ &= y_t^* + \left[\eta + (\frac{1}{\sigma}-\eta)(1-\alpha')\left(1-\frac{\alpha}{1-\kappa}\Phi_{SS2}^{-1}\right)\right] s_t \qquad (5.164) \\ &= y_t^* + \frac{\omega}{\sigma}s_t\,, \qquad (5.165) \end{aligned}$$

where $\omega \equiv \sigma\eta + (1-\sigma\eta)(1-\alpha')\left(1-\frac{\alpha}{1-\kappa}\Phi_{SS2}^{-1}\right)$. Notice that in the case of zero trade costs, ω equals the parameter ω_α in GM, and the last equation simplifies to

$$y_t = y_t^* + \frac{\omega_\alpha}{\sigma}s_t\,, \quad \omega_\alpha \equiv 1 + \alpha(2-\alpha)(\sigma\eta - 1) > 0\,.$$

As a sixth step, one can use the consumption ratio given in Equation (4.32), substitute out s_t and get an equation that relates c_t to domestic and world output:

$$c_t = \Phi_c y_t + (1-\Phi_c)y_t^*\,, \qquad (5.166)$$

where the parameter $\Phi_c \equiv \frac{1-\alpha'}{\omega}$. In the seventh step, Equation (5.166) is used to replace consumption in the household's Euler Equation (4.30), and first differences of Equation (4.20) is used to replace CPI inflation by domestic goods inflation:

$$\Phi_c y_t + (1-\Phi_c)y_t^* = E_t\left\{\Phi_c y_{t+1} + (1-\Phi_c)y_{t+1}^*\right\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{H,t+1} + \alpha'\Delta s_{t+1}\})\,. \quad (5.167)$$

Finally, the eighth and last step is to substitute out Δs_{t+1} using equation (5.165) and to solve for y_t. One then obtains a dynamic IS equation for the small open economy:

$$y_t = E_t\{y_{t+1}\} - \frac{\omega}{\sigma}(r_t - E_t\{\pi_{H,t+1}\}) + (\omega - 1)\,E_t\{\Delta y_{t+1}^*\}\,. \qquad (5.168)$$

To derive the New Keynesian Phillips Curves, I start from Equation (4.35) derived in this appendix Section 5.3.6. The marginal costs in these equations shall be replaced by output. Remember from Section 4.2.2, that $MC_t^n = MC_t P_{H,t} = (1-\tau)W_t/A_t$, so the log deviation of the real marginal costs of the small open and the world economy are

$$\widehat{mc}_t = w_t - a_t - p_{H,t} \quad \text{and} \quad \widehat{mc}_t^* = w_t^* - a_t^* - p_t^*. \qquad (5.169)$$

For the world economy, the household's intratemporal first-order condition $w_t^* - p_t^* =$

$\sigma c_t^* + \varphi n_t^*$ and aggregate production $y_t^* = n_t^* + a_t^*$, analogously to Equations (4.30) and (4.33), can be used to rewrite

$$\widehat{mc}_t^* = (\sigma + \varphi) y_t^* - (1 + \varphi) a_t^* \,. \tag{5.170}$$

For the small open economy, the same steps and additionally Equation (4.20) result in

$$\widehat{mc}_t = \sigma c_t + \varphi y_t + \alpha' s_t - (1 + \varphi) a_t. \tag{5.171}$$

Now, using Equation (4.32) allows for replacing consumption by world output and terms of trade,

$$\widehat{mc}_t = \sigma y_t^* + \varphi y_t + s_t - (1 + \varphi) a_t. \tag{5.172}$$

Finally, Equation (5.165) enables us to substitute out s_t. So marginal costs can be rewritten just in terms of both types of output and domestic productivity:

$$\widehat{mc}_t = \left(\frac{\sigma}{\omega} + \varphi\right) y_t + \sigma \left(1 - \frac{1}{\omega}\right) y_t^* - (1 + \varphi) a_t. \tag{5.173}$$

To use the conventional notation in terms of gaps, the output gap shall be defined as the deviation of the log-linearized variable from its natural level, which would occur under flexible prices and thereby constant marginal costs $\log MC_t = mc_t = \log MC_t^* = mc_t^* = -\mu$. This implies that the log deviations of marginal costs from this flex-price steady state are always zero, $\widehat{mc}_t = \widehat{mc}_t^* = 0$. Thus, I have $\tilde{y}_t \equiv y_t - \overline{y}_t$ and analogously $\tilde{y}_t^* \equiv y_t^* - \overline{y}_t^*$, where bars above variables with time index are used to denote their natural levels. To obtain these natural levels of output, solve Equations (5.173) and (5.170) in the flex-price situation for the respective output:

$$\overline{y}_t = \frac{\omega(1 + \varphi)}{\sigma + \omega\varphi} a_t + \frac{\sigma(1 - \omega)}{\sigma + \omega\varphi} y_t^* \quad \text{and} \quad \overline{y}_t^* = \frac{1 + \varphi}{\sigma + \varphi} a_t^*. \tag{5.174}$$

Subtracting the flex-price version of Equation (5.170) from the sticky price version yields

$$\begin{aligned} \widehat{mc}_t^* &= (\sigma + \varphi)(y_t^* - \bar{y}_t^*) \\ &= (\sigma + \varphi) \tilde{y}_t^* \,. \end{aligned} \tag{5.175}$$

Similarly, for the small open economy we obtain

$$\begin{aligned}\widehat{mc}_t &= \left(\frac{\sigma}{\omega}+\varphi\right)(y_t-\bar{y}_t)\\ &= \left(\frac{\sigma}{\omega_\xi}+\varphi\right)\tilde{y}_t\,. \end{aligned} \tag{5.176}$$

Notice that foreign output does not show up, as for the calculation of the domestic output gap world output is assumed to be exogenous, both in the flex-price and in the sticky price world.

After inserting the results for marginal costs from Equations (5.175) and (5.176) in the inflation dynamics equations given in (4.35), I obtain the New Keynesian Phillips curves (NKPC) for the small open economy and for the world economy, linking inflation to its expected future value and to the output gap:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \Phi_{NKPC}\tilde{y}_t\,, \tag{5.177}$$

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \Phi_{NKPC^*}\tilde{y}_t^*\,, \tag{5.178}$$

where $\Phi_{NKPC} \equiv \lambda\left(\frac{\sigma}{\omega}+\varphi\right)$ and $\Phi_{NKPC^*} \equiv \lambda(\sigma+\varphi)$.

For the dynamic IS equations, start with the difference equation for world output given in equation (5.156). Evaluate it twice, once for sticky prices and once for flexible prices. In doing so, notice that

$$\bar{r}_t^* - E_t\{\bar{\pi}_{t+1}^*\} = -\sigma(1-\rho_a^*)\Gamma_0 a_t^* \equiv \overline{rr}_t^*. \tag{5.179}$$

is the natural expected real rate of interest in the world economy, which would prevail under completely flexible prices. It can be derived by solving Equation (5.156) for the flexible price situation characterized by equation (5.174). Subtract the flex-price outcome from the sticky price outcome to obtain

$$\tilde{y}_t^* = E_t\{\tilde{y}_{t+1}^*\} - \frac{1}{\sigma}(r_t^* - E_t\{\pi_{t+1}^*\} - \overline{rr}_t^*)\,. \tag{5.180}$$

Analogously, the small open economy's dynamic IS equation is obtained by subtracting Equation (5.174) from Equation (5.168) and simplifying:

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{\omega}{\sigma}(r_t - E_t\{\pi_{H,t+1}\} - \overline{rr}_t) \tag{5.181}$$

with the domestic natural expected real rate of interest

$$\overline{rr}_t \equiv -\frac{\sigma(1+\varphi)(1-\rho_a)}{\sigma+\omega\varphi}a_t - \varphi\frac{\sigma(1-\omega)}{\sigma+\omega\varphi}E_t\{\Delta y^*_{t+1}\}, \tag{5.182}$$

again derived evaluating Equation (5.168) at the flexible price situation described by equation (5.174). Equations (5.177), (5.178), (5.181) and (5.180) are equations (4.38), (4.39), (4.40) and (4.41) in Section 4.2.3.

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Ich erkläre hiermit, dass ich die vorliegende Dissertation mit dem Titel "Essays on Macroeconomic Theory as a Guide to Economic Policy´´ selbständig und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe; die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sind als solche kenntlich gemacht.
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Berlin, den 9. April 2009 Stefan Ried